Mutations of quiver with potential at several vertices

Laurent Demonet
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Introduction

Mutations of quivers with potential (Derksen, Weyman, Zelevinsky)

Categorification of every skew-symmetric cluster algebra.
Interpretation of F-polynomials, $\mathbf{g}$-vectors in this context $\Rightarrow$ proof of important combinatorial conjectures of Fomin and Zelevinsky.

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Question

Explicit formula for mutation of quivers with potentials at several vertices?
Partial answer by Keller's green sequences.

Notation

$(Q, W) = (Q_0, Q_1, W)$: a quiver with potential.
$R = kQ_0$. $A = kQ_1$. A is a R-bimodule.

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$$\Lambda = J(Q, W) = \frac{\widehat{kQ}}{\partial_{A^*} W} = \frac{R\langle\langle A\rangle\rangle}{\partial_{A^*} W}$$

- $\partial W : A^* \to k_Q$;
- $\partial_{A^*} W$ is the complete ideal generated by $\operatorname{im} \partial W$.

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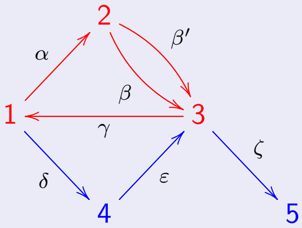
Aim

Computing $\mu_K(Q, W)$.

Mutation

Example



$$W = \alpha\beta\gamma - \gamma\delta\varepsilon$$

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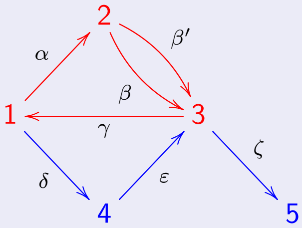

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\end{array}$$

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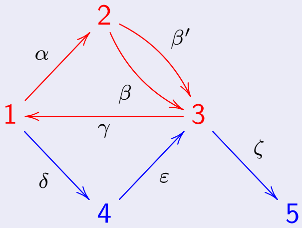

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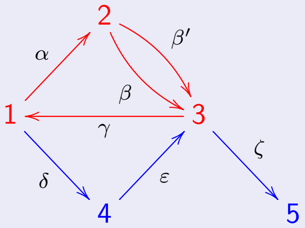

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Remark: $X = \Lambda_K \otimes_\Lambda {}_\Lambda \mathrm{rad}(\Lambda e_{\overline{K}})$

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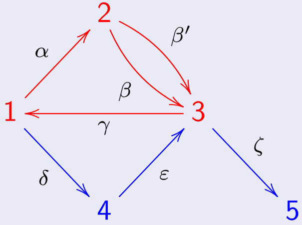

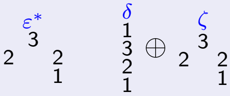

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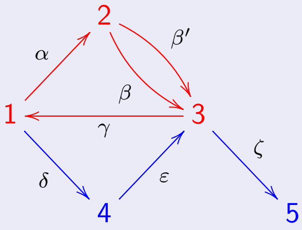

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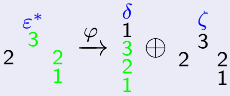

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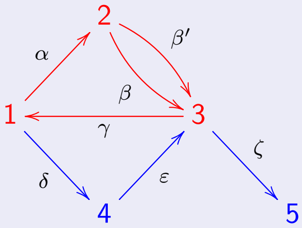

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Remark

Exact in degree -1. $\operatorname{coker} \gamma \simeq \Lambda_K$.
Exact if Λ_K is bimodule 3-Calabi-Yau.

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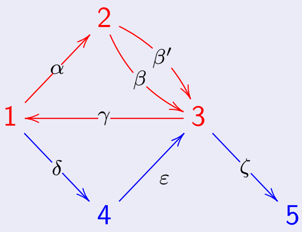

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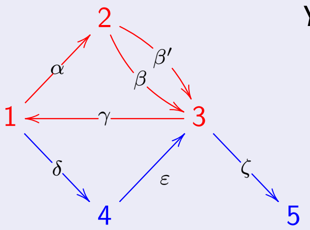

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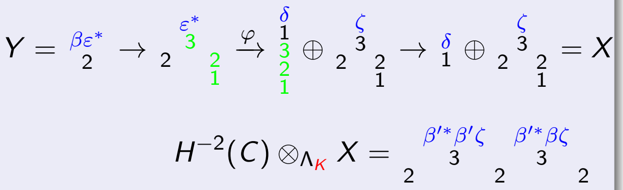

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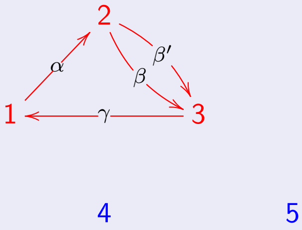

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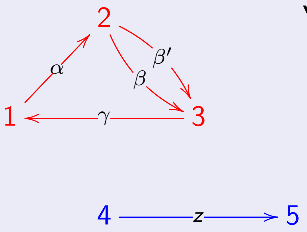

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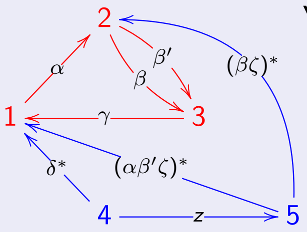

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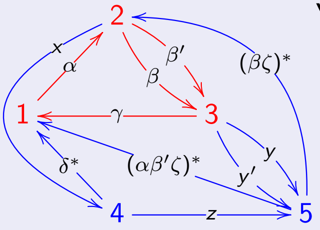

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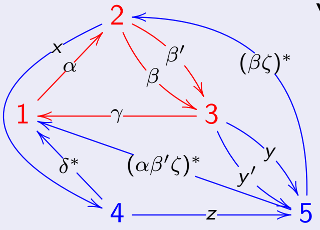

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Remark: green sequence: $1, 3, 2, 1$ followed by $1 \leftrightarrow 2$.

Mutation

Green sequence [applet by B. Keller]

$${}_{K}\widetilde{A}_{K} = {}_{K}A_{K}$$

$${}_{\overline{K}}\widetilde{A}_{K} = \mathrm{top}_{K}(X^{*})$$

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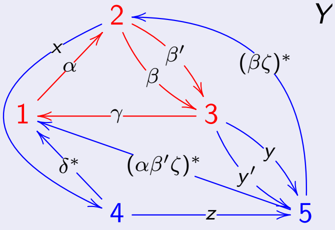

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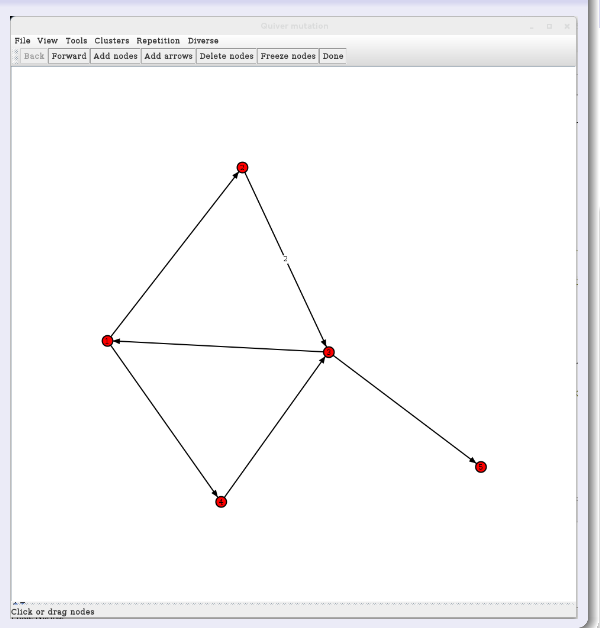

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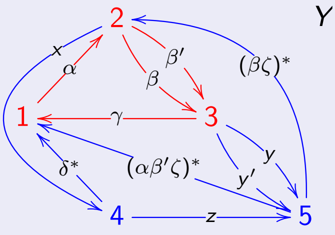

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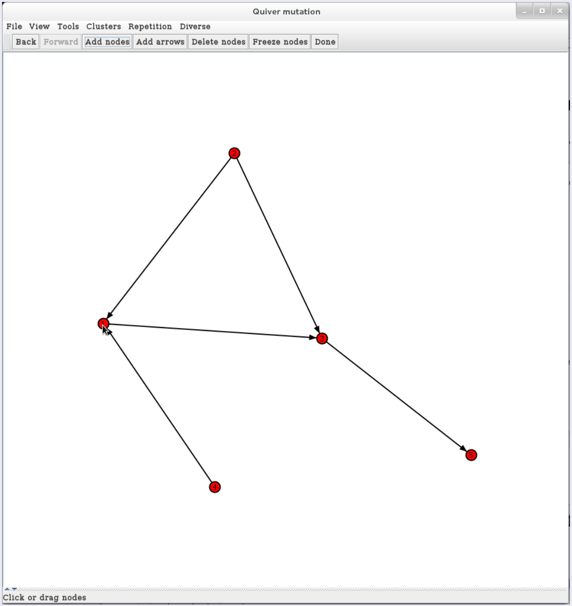

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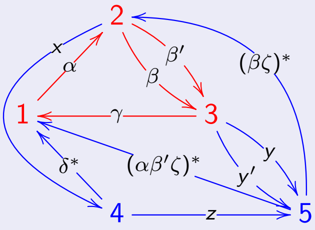

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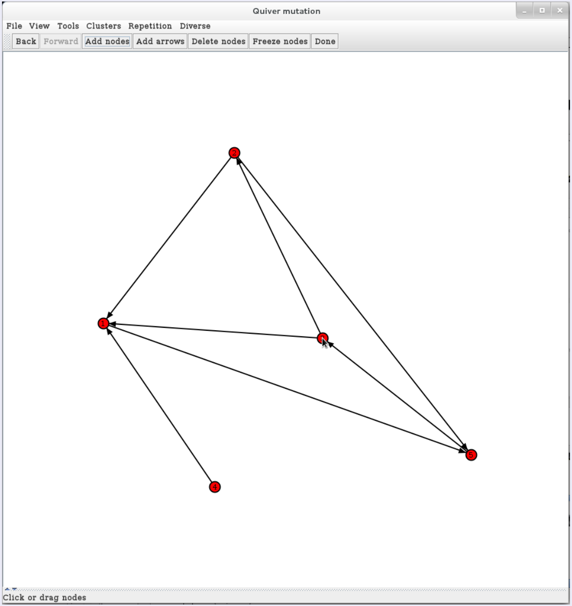

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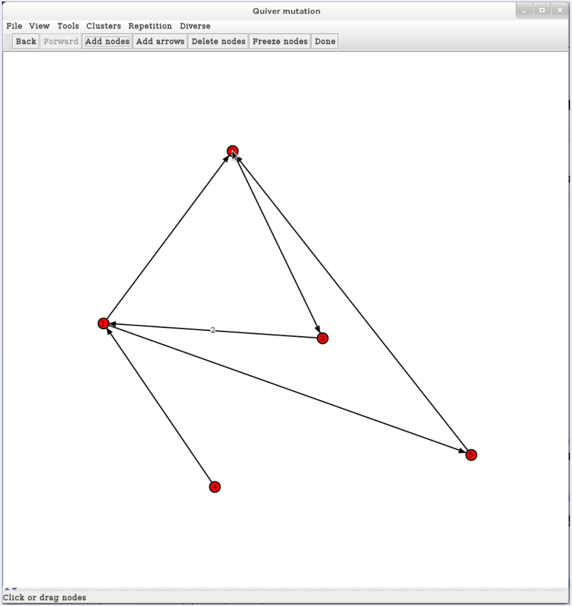

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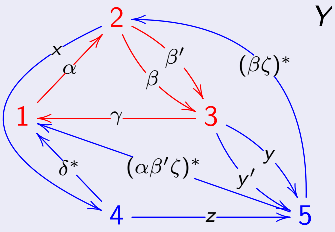

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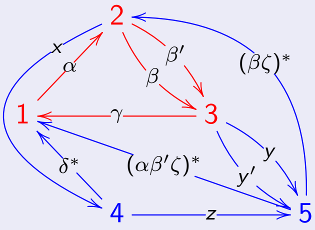

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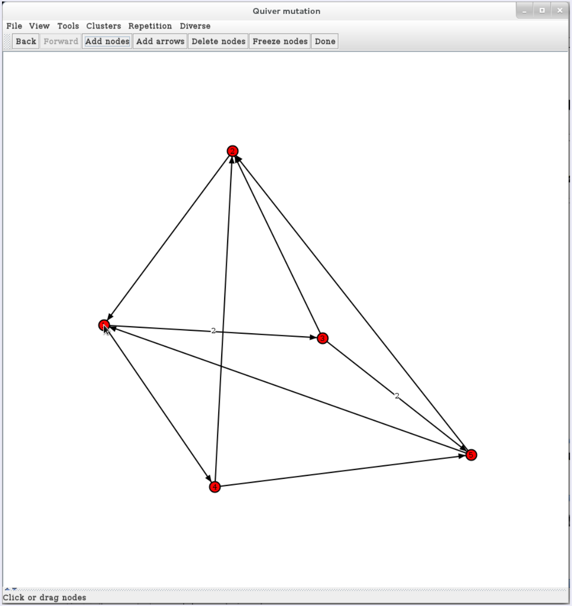

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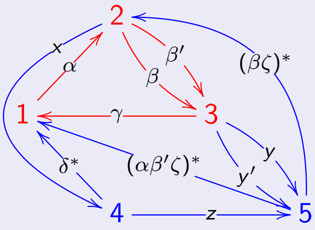

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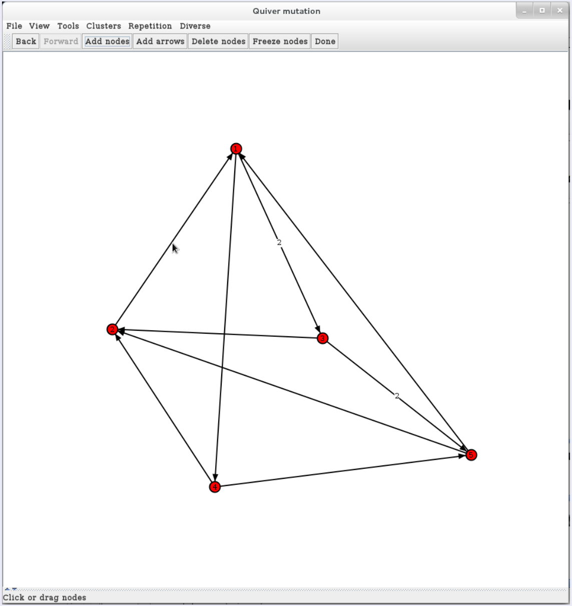

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$$ {}_{K}\widetilde{A}_{K} = {}_{K}A_{K}; \quad {}_{\overline{K}}\widetilde{A}_{\overline{K}} = {}_{\overline{K}}A_{\overline{K}} \oplus \mathrm{Hom}_{\Lambda_K}(Y, X); $$

$$ {}_{\overline{K}}\widetilde{A}_{K} = \mathrm{top}_K\,(X^*)\,; \quad {}_{K}\widetilde{A}_{\overline{K}} = {}_{K}\mathrm{top}\left(H^{-2}(C)\otimes_{\Lambda_K} X \oplus Y\right) $$

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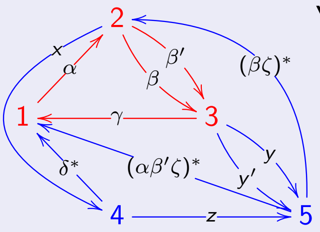

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1. *If* $\#K = 1$, $\mu_K^\pm$ *is the DWZ mutation.*
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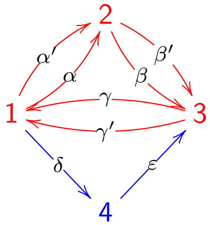
4. *Corresponds to maximal green sequences result in explicitely computed cases.*

Another example

$u, v \neq 0$



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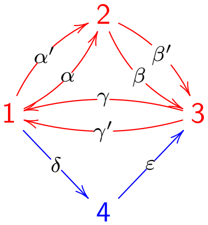

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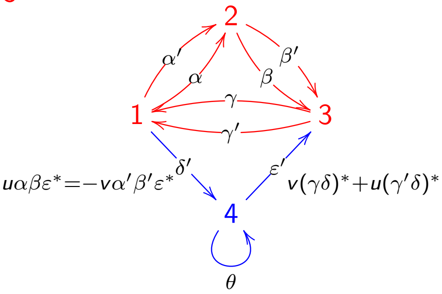

$$\widetilde{W} = \alpha\beta\gamma + \alpha'\beta'\gamma' - \alpha\beta'\gamma\alpha'\beta\gamma' - (u\gamma + v\gamma')\delta'\varepsilon' + (v\gamma - u\gamma')\delta'\theta\varepsilon'$$