

# One parameter 3-equipped posets

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Alexander G. Zavadskij introduced partially ordered sets with an order relation of two kinds and called them equipped posets (actually we call them 2-equipped, in the sense of generalized equipped posets).

A.V. Zabari and A.G. Zavadskij, *One-parameter equipped posets and their representations*. Functional. Anal.i Prilozhen., **34** (2000), no. 2, 72-75 (Russian), English transl. in Functional Anal. Appl., **34** (2000), no.2, 138-140.

Their representations are modules over certain artinian shurian algebras over a non-algebraically closed field. They determine some matrix problems of mixed type over a quadratic field extension.

A.G. Zavadskij, *Tame equipped posets*. Linear Algebra and its Appl., Vol. **365** (2003), 389-465.

A.G. Zavadskij, *Equipped posets of finite growth*. Representations of Algebras and Related Topics, AMS, Fields Inst. Comm. Ser., Vol. **45**(2005), 363-396.

C. Rodriguez and A.G. Zavadskij, *On corepresentations of equipped posets and their differentiation*. Revista Colombiana de Matematicas, **41** (2007), 117-142.

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- ▶ Definition of 3-equipped posets, and their representations and corepresentations over an inseparable cubic field extension  $F \subset G$ , in characteristic 3.

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- ▶ Properties of dimension vectors of indecomposable representations and corepresentations of three-equipped posets, in terms of the Tits quadratic form.

# 3-equipped poset

## Definition

A finite poset  $(\mathcal{P}, \leq)$  is called **3-equipped** if to every comparable pair of its points  $x \leq y$ , we assign one and only one value

$$x \leq^1 y \quad \text{or} \quad x \leq^2 y \quad \text{or} \quad x \leq^3 y$$

and the following condition holds:

If  $x \leq^l y \leq^m z$  and  $x \leq^n z$ , then  $n \geq \min\{lm, 3\}$ .

# 3-equipped poset

We use the terminology:

if  $x \leq^1 y$  we say that  $x$  and  $y$  are in **weak** relation,

if  $x \leq^2 y$  we say that  $x$  and  $y$  are in **semistrong** relation,

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For every point  $x \in \mathcal{P}$  we have  $x \leq^1 x$ , or  $x \leq^3 x$ .

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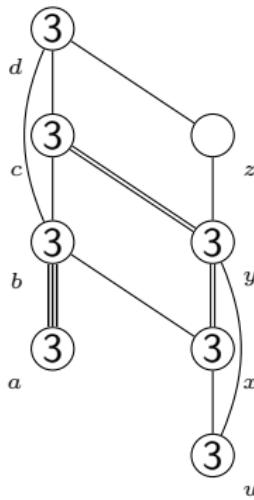
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Any relation between a strong point and an arbitrary point is always strong.

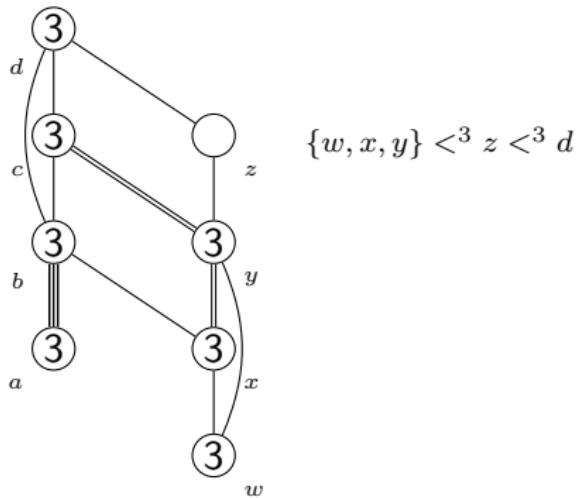
# Ordinary posets as 3-equipped posets

A 3-equipped poset  $(\mathcal{P}, \leq)$  is called **trivially equipped** if it contains only strong points; in this case  $(\mathcal{P}, \leq)$  is an **ordinary** poset.

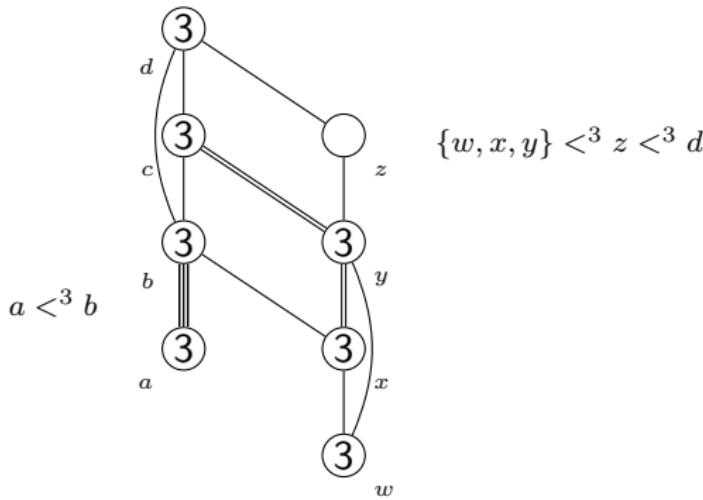
# Example of a 3-equipped poset



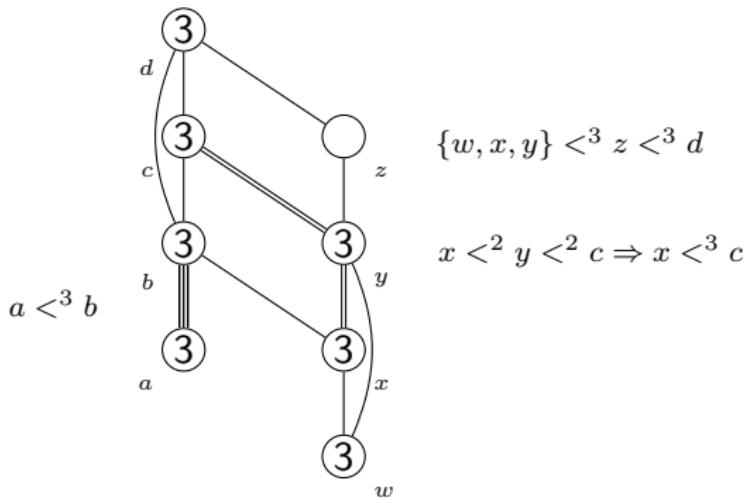
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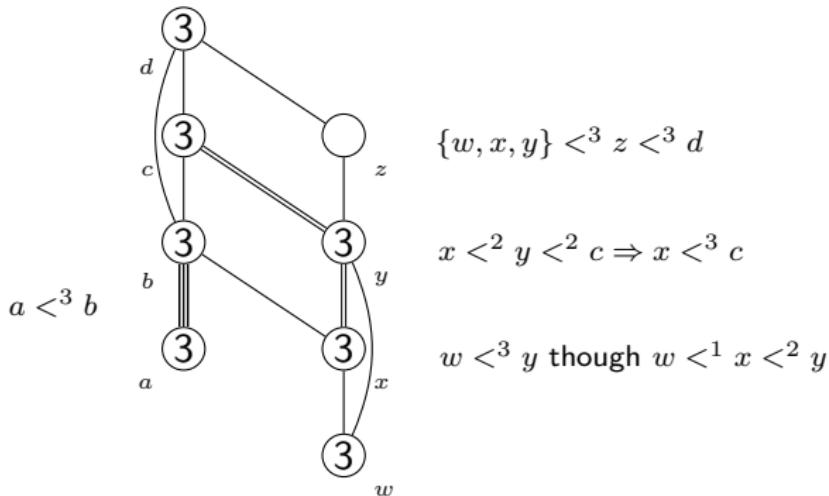
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# Representations and corepresentations

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$G = F(\xi)$ .

## A natural derivation over the field G

The derivation  $\delta$  over the field G, is the application  $\delta : G \rightarrow G$  such that

$$\delta(a) = 0, \text{ for every } a \in F, \quad \delta(\xi) = 1, \quad \delta(\xi^2) = -\xi.$$

For each  $g \in G$ , we denote  $g^\delta = \delta(g)$ .

As  $g \in G$  can be written  $g = a + b\xi + c\xi^2$  for some  $a, b, c \in F$ , notice that  $g^\delta = b - c\xi$ .

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$\delta$  appears in a natural way when we are solving the matrix problem corresponding to some critical 3-equipped poset.

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# Representations of 3-equipped posets

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Two representations  $U$  and  $V$  are isomorphic ( $U \simeq V$ ) if and only if there exists a  $\mathbb{F}$ -vector space isomorphism  $\varphi : U_0 \rightarrow V_0$  such that  $(\varphi \otimes 1)(U_x) = V_x$  for all  $x \in \mathcal{P}$ .

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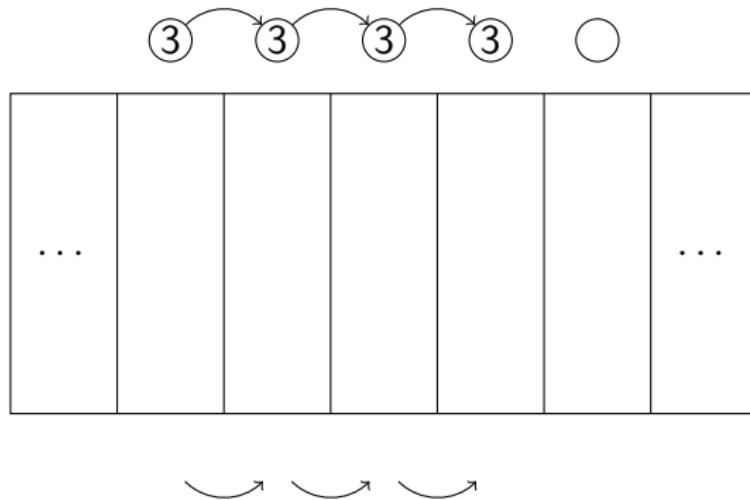
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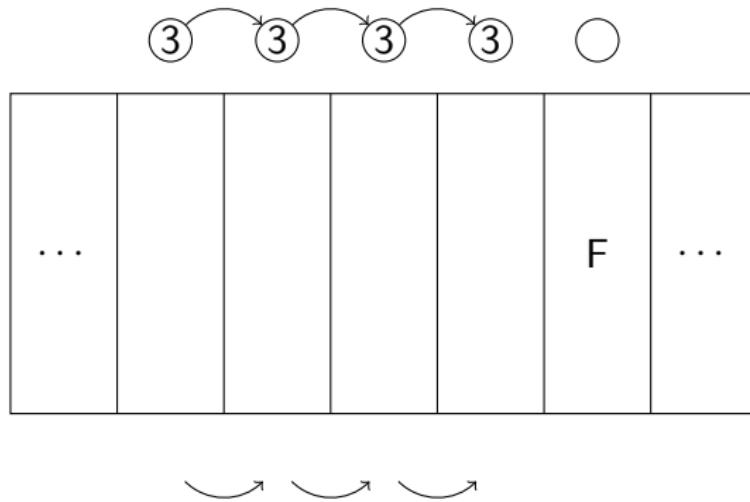
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We apply the following **admissible transformations** to  $M$ .

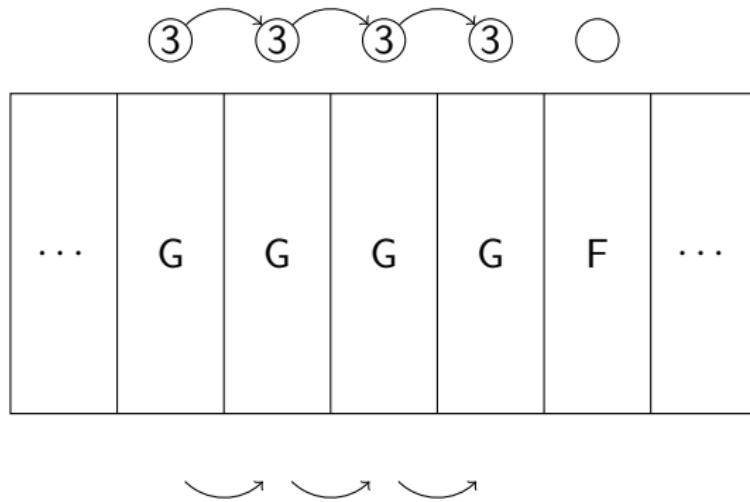
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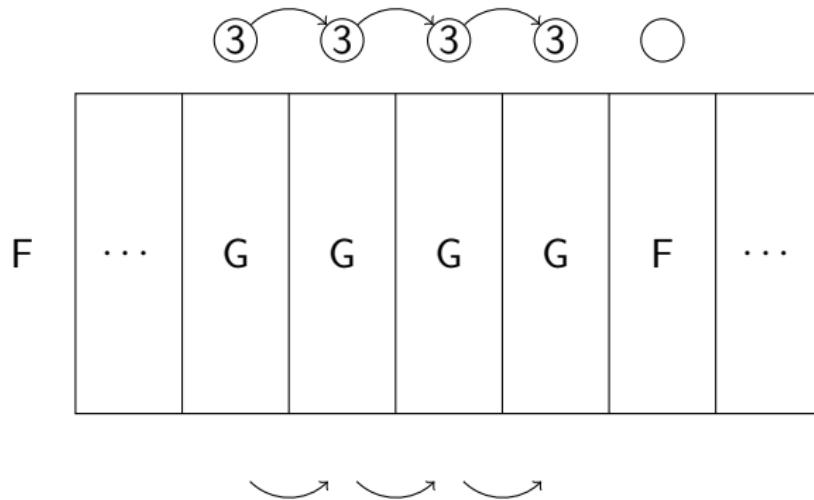
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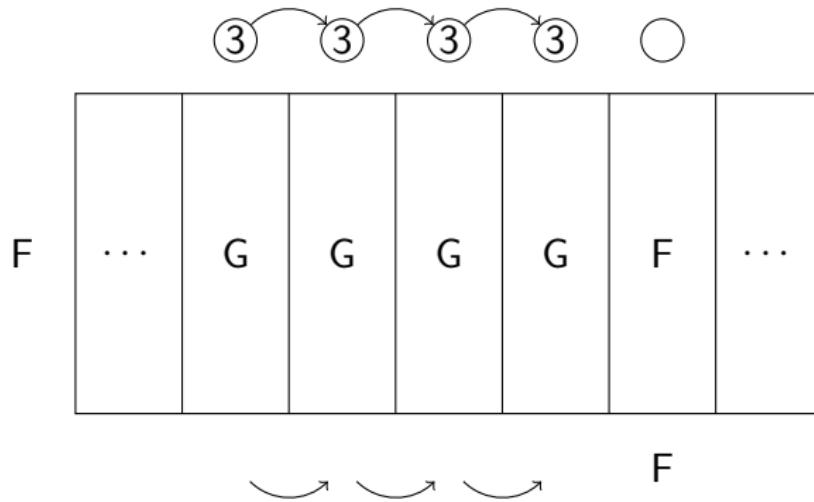
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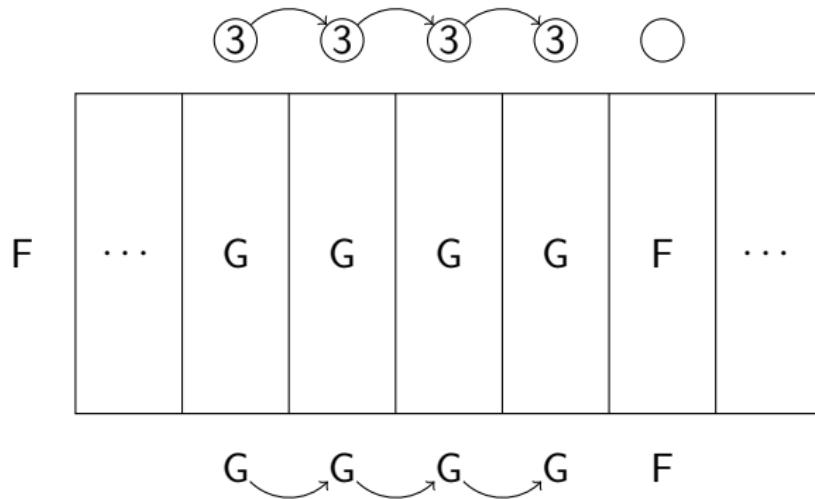
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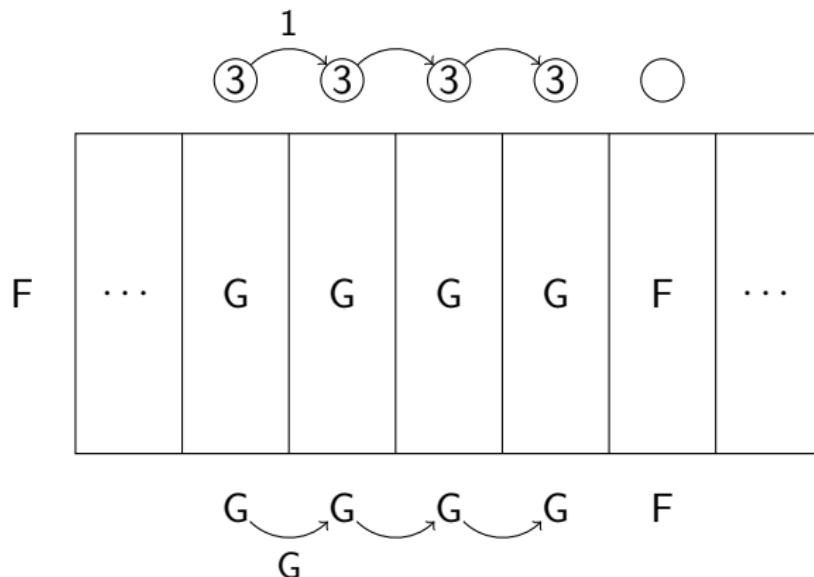
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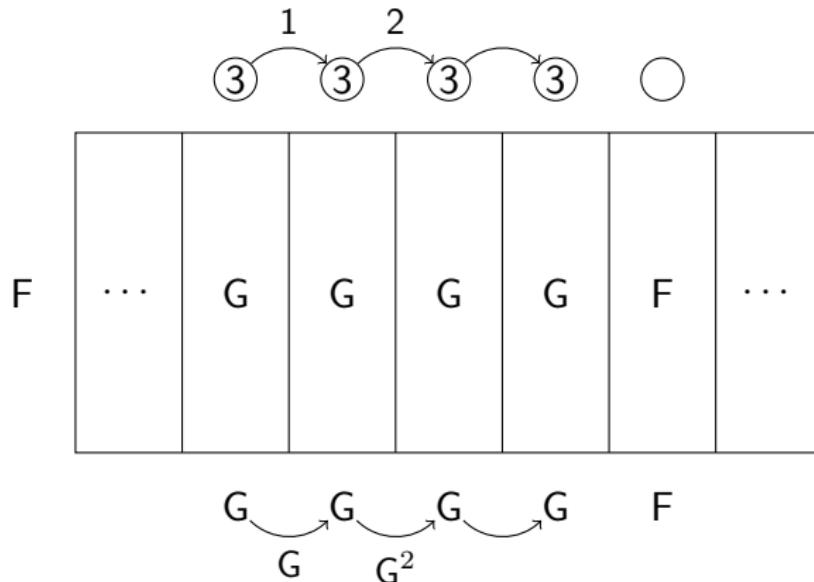
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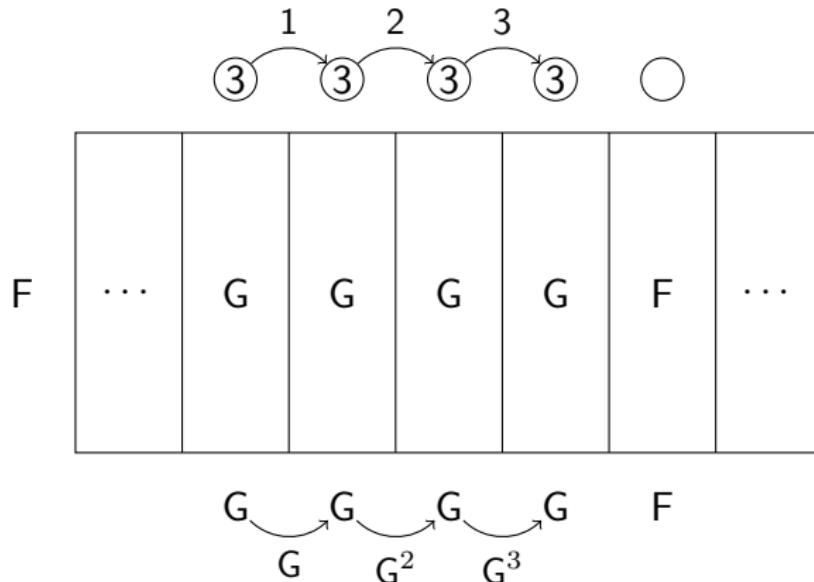
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## Corepresentations of 3-equipped posets

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Two corepresentations  $U$  and  $V$  are **isomorphic** ( $U \simeq V$ ) if and only if there is a G-isomorphism  $\varphi : U_0 \rightarrow V_0$  such that  $\varphi(U_x) = V_x$  for all  $x \in \mathcal{P}$ .

# Matrix corepresentation

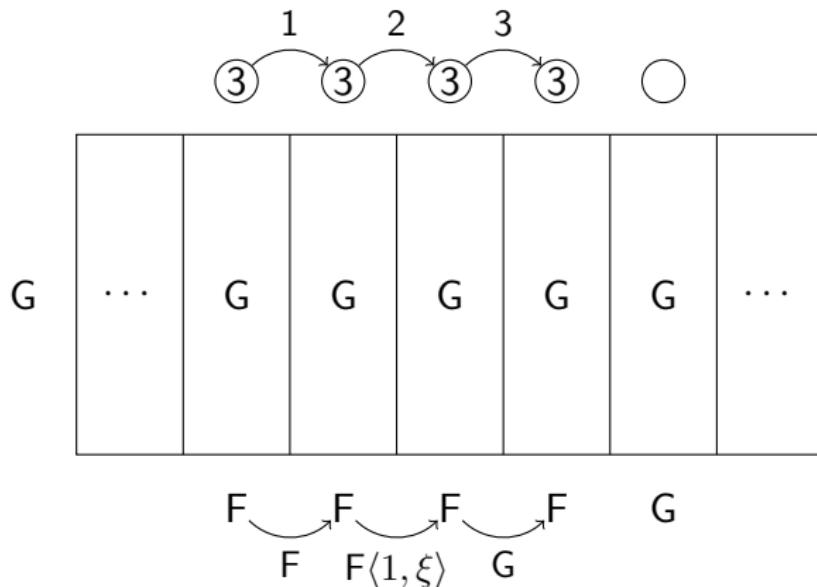
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Its **admissible transformations** are the following

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# One parameter 3-equipped posets

# Series

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Each representation over  $(F[t], G[t])$  generates an **F-series** of representations over  $(F, G)$  by substituting a square matrix  $A$  (in a standard canonical form) with values in  $F$  for the variable  $t$ , and scalar matrices  $gI$  of the same size, for each scalar element  $g \in G$ .

## Example of an F-series

For the poset  $K_{10}$



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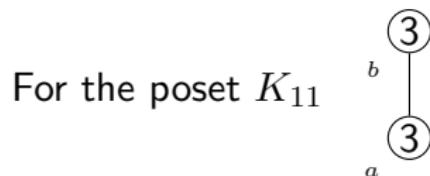
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where  $A$  is a matrix over  $\mathbb{F}$ , of order  $n$  in a standard canonical form with respect to ordinary similarity transformations ( $X^{-1}AX$ ) over  $\mathbb{F}$ .

# Series

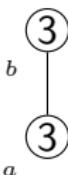
If  $M$  is a  $(F[t], G[t])$ -representation such that the variable  $t$  does not appear in the stripes corresponding to the strong points, then  $M$  generates a **G-series** of representations over  $(F, G)$  by substituting a square matrix  $A$  (in a standard canonical form) with values in  $G$  for the variable  $t$ , and scalar matrices  $gI$  of the same size, for each scalar element  $g \in G$ .

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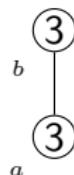


The Hasse diagram of the poset  $K_{11}$  shows two nodes. The top node is labeled  $b$  and contains the number 3 in a circle. The bottom node is labeled  $a$  and also contains the number 3 in a circle. A vertical line connects the two nodes, with the letter  $b$  positioned above the line and the letter  $a$  positioned below it.

$a$	$b$
1	0
$\xi$	1
$\xi^2$	$t$

## Example of a G-series

For the poset  $K_{11}$



the representation

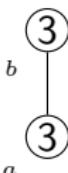
$a$	$b$
1	0
$\xi$	1
$\xi^2$	$t$

generates the G-series

$I$	0
$\xi I$	$I$
$\xi^2 I$	$A$

## Example of a G-series

For the poset  $K_{11}$



the representation

$a$	$b$
1	0
$\xi$	1
$\xi^2$	$t$

generates the G-series

$a$	$b$
$I$	0
$\xi I$	$I$
$\xi^2 I$	$A$

where  $A$  is a matrix over  $G$ ,

of order  $n$  in a standard canonical form with respect to the following pseudolinear similarity transformations over  $G$ ,

$$X^{-1}AX + X^{-1}X^\delta$$

## Corepresentation series

Every matrix corepresentation over the pair of polynomial rings  $(F[t], G[t])$  generates an **F(G)-series** of corepresentations over  $(F, G)$  by substituting a square matrix  $A$  (in a standard canonical form) with values in  $F$  ( $G$ ) for the variable  $t$ , and scalar matrices  $gI$  of the same size, for each scalar element  $g \in G$ .

# One parameter 3-equipped posets

A 3-equipped poset  $(\mathcal{P}, \leq)$  is **one parameter**, if it is of infinite type and there exists one series, in each dimension, containing almost all its indecomposable representations or corepresentations (up to isomorphism).

# One parameter criterion for 3-equipped posets

## THEOREM 1

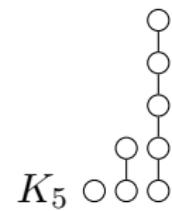
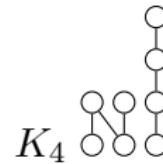
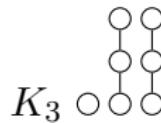
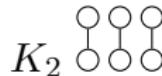
Let  $F$  be a field of characteristic 3,  $G$  be an inseparable cubic extension over  $F$ , and  $(\mathcal{P}, \leq)$  be a 3-equipped poset. Then the following statements are equivalent:

1.  $\mathcal{P}$  is one parameter with respect to representations and corepresentations.
2.  $\mathcal{P}$  contains exactly one of the critical 3-equipped posets  $K_1, \dots, K_5, K_{10}, K_{11}$  as a subposet, and does not contain the poset  $W_{10}$  of the form

③ ③

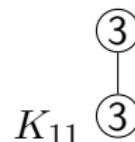
# Critical 3-equipped posets

The first five posets in the list are the Kleiner's critical posets.



# Critical 3-equipped posets

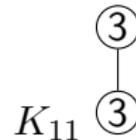
The last two are the (non-trivially equipped) **critical 3-equipped posets**.



# Sincere one parameter 3-equipped posets

## THEOREM 2

The only sincere one parameter 3-equipped posets, which are not ordinary, are the critical  $K_{10}$  and  $K_{11}$ .



# Indecomposables of one parameter 3-equipped posets

## THEOREM 3

The sincere indecomposables of the critical 3-equipped poset  $K_{10}$  are exhausted, up to isomorphism, by:

- (I) 33 kinds of pairwise non-isomorphic sincere indecomposable matrix representations.
- (II) 33 kinds of pairwise non-isomorphic sincere indecomposable matrix corepresentations.

# Indecomposables of one parameter 3-equipped posets

## THEOREM 4

The sincere indecomposables of the critical 3-equipped poset  $K_{11}$  are exhausted, up to isomorphism, by:

- (I) 29 kinds of pairwise non-isomorphic sincere indecomposable matrix representations.
- (II) 29 kinds of pairwise non-isomorphic sincere indecomposable matrix corepresentations.

# Representations of $K_{10}$

One series

$$f = 0$$

$I$	$I$
$\xi I$	$A$

where  $A$  is a matrix of order  $n$  in a standard canonical form with respect to similarity  $(X^{-1}AX)$  over  $\mathbb{F}$ .

# Representations of $K_{10}$

$$f = 0$$

1	0	0	1	0	0
$\xi$	1	0	0	0	0
$\xi^2$	$-\xi$	0	0	0	0
0	0	1	1	0	0
0	0	$\xi$	0	1	0
0	0	$\xi^2$	0	0	1

# Representations of $K_{10}$

$f = 3$				
1	0	1	0	0
$\xi$	1	0	1	0
$\xi^2$	$-\xi$	0	0	1

$f = 3$						
1	0	0	0	1	0	0
0	1	0	0	0	1	0
$\xi$	0	1	0	0	0	1
0	$\xi$	0	1	0	0	0
$\xi^2$	0	$-\xi$	0	0	0	0
0	$\xi^2$	0	$-\xi$	0	0	1

# Representations of $K_{10}$

$f = 3$

1	0	1	0	0
0	1	0	1	0
$\xi$	0	0	0	1
0	$\xi$	0	0	0
$\xi^2$	0	0	0	0
0	$\xi^2$	0	0	1

$f = 3$

1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
$\xi$	0	0	0	0	0	1
0	$\xi$	0	0	0	1	0
0	0	$\xi$	0	1	0	0
0	0	0	1	1	0	0
0	0	0	$\xi$	0	1	0
0	0	0	$\xi^2$	0	0	1

# Corepresentations of $K_{10}$

One series

$$\widehat{f} = 0$$

$I$	$\xi I$	$\xi^2 I$	$I$
$0$	$I$	$-\xi I$	$A$

where  $A$  is a matrix of order  $n$  in a stander canonical form with respect to pseudolinear  $(I + X^{-1}AX^\delta)$  over  $G$ .

# Representations of $K_{11}$

## Series

$$f = 0$$

$I$	$0$
$\xi I$	$I$
$\xi^2 I$	$-\xi I + A$

where  $A$  is a matrix of order  $n$  in a standard canonical form with respect to pseudolinear  $(I + X^{-1}AX^\delta)$  over  $G$ .

# Corepresentations of $K_{11}$

## Series

$$\begin{array}{c} \hat{f} = 0 \\ \boxed{I \quad \xi I + \xi^2 A} \end{array}$$

where  $A$  is a matrix of order  $n$  in a stander canonical form with respect to similarity  $(X^{-1}AX)$  over  $\mathbb{F}$ .

# Corepresentations of $K_{11}$

$$\widehat{f} = 0$$

1	$\xi$	$\xi^2$	0	0	0
0	1	$-\xi$	0	0	1
0	0	0	1	$\xi$	$\xi^2$

# Corepresentations of $K_{11}$

$$\widehat{f} = 3$$

1	$\xi$	$\xi^2$	0	0	0
0	1	$-\xi$	1	$\xi$	$\xi^2$

$$\widehat{f} = 3$$

1	0	$\xi$	0	$\xi^2$	0	0	0	0
0	1	0	$\xi$	0	$\xi^2$	0	0	0
0	0	1	0	$-\xi$	0	1	$\xi$	$\xi^2$
0	0	0	1	0	$-\xi$	0	1	$-\xi$

$$\widehat{f} = 3$$

1	0	0	0	0	$\xi$
0	1	0	0	$\xi$	0
0	0	1	$\xi$	0	0
0	0	0	1	$\xi$	$\xi^2$

$$\widehat{f} = 3$$

1	0	0	0	0	$\xi$	0	$\xi$
0	1	0	0	$\xi$	0	0	0
0	0	1	$\xi$	0	0	0	0
0	0	0	1	0	$\xi$	0	$\xi^2$
0	0	0	0	1	0	$\xi$	0

# Corepresentations of $K_{11}$

$$\widehat{f} = 1$$

1	$\xi$	$\xi^2$
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$$\widehat{f} = 1$$

1	$\xi$	$\xi^2$	0
0	1	$-\xi$	1

$$\widehat{f} = 1$$

1	$\xi$	$\xi^2$	0	0
0	1	$-\xi$	1	$\xi$

$$\widehat{f} = 1$$

1	$\xi$	0
0	1	$\xi$

$$\widehat{f} = 1$$

1	$\xi$	0	0
0	1	$\xi$	$\xi^2$

$$\widehat{f} = 1$$

1	0	0	$\xi$	0
0	1	$\xi$	0	0
0	0	1	$\xi$	$\xi^2$

## Possible applications

- ▶ Application to representations of vector space categories (vectroids).
- ▶ Application to representations of artinian schurian right peak rings (PI rings).

I. Dorado, *Three-equipped posets and their representations and corepresentations (inseparable case)*. Linear Algebra and its Applications, Vol. **433**(2010), 1827-1850.

Thank you