From trisections in module categories to quasi-directed components

Flávio Ulhoa Coelho - IME-USP

ICRA 2012 - Bielefeld

13 de agosto de 2012

- $\Lambda$ - basic and connected artin algebra
- $\text{mod } \Lambda$ - category of finitely generated right $\Lambda$-modules
- $\text{ind } \Lambda$ - full subcategory of $\text{mod } \Lambda$ consisting of exactly one representative from each isomorphism class of indecomposable $\Lambda$-modules.
- $\Gamma (\text{mod } \Lambda)$ - the Auslander-Reiten quiver of $\Lambda$. 
Preliminaries


- $\Lambda$ - basic and connected artin algebra
- $\text{mod } \Lambda$ - category of finitely generated right $\Lambda$-modules
- $\text{ind } \Lambda$ - full subcategory of $\text{mod } \Lambda$ consisting of exactly one representative from each isomorphism class of indecomposable $\Lambda$-modules.
- $\Gamma (\text{mod } \Lambda)$ - the Auslander-Reiten quiver of $\Lambda$. 
A generalised standard component $\Gamma \subset \Gamma(\mod \Lambda)$ is **quasi-directed** provided there exist at most finitely many modules lying in oriented cycles.

Such components appear naturally in the so-called *laura algebras* (introduced by Assem-C. [AC] and Reiten-Skowroński [RS]).
A generalised standard component $\Gamma \subset \Gamma(\text{ mod } \Lambda)$ is **quasi-directed** provided there exist at most finitely many modules lying in oriented cycles.

Such components appear naturally in the so-called *laura algebras* (introduced by Assem-C. [AC] and Reiten-Skowroński [RS]).
\[ \mathcal{L}_\Lambda = \{ X \in \text{ind}\Lambda : \text{pd} Y \leq 1 \ \forall Y \ \text{predecessor of } X \} \]

\[ \mathcal{R}_\Lambda = \{ X \in \text{ind}\Lambda : \text{id} Y \leq 1 \ \forall Y \ \text{successor of } X \} \]

**Definition**

An algebra \( \Lambda \) is called **laura** if \( \mathcal{L}_\Lambda \cup \mathcal{R}_\Lambda \) is cofinite in \( \text{ind} \Lambda \).
Laura algebras

\[ \mathcal{L}_\Lambda = \{ X \in \text{ind}\Lambda : \text{pd} Y \leq 1 \ \forall Y \text{ predecessor of } X \} \]

\[ \mathcal{R}_\Lambda = \{ X \in \text{ind}\Lambda : \text{id} Y \leq 1 \ \forall Y \text{ successor of } X \} \]

**Definition**

An algebra \( \Lambda \) is called laura if \( \mathcal{L}_\Lambda \cup \mathcal{R}_\Lambda \) is cofinite in \( \text{ind} \Lambda \).
Laura algebras

\[ \mathcal{L}_\Lambda = \{ X \in \text{ind}\Lambda : \text{pd} Y \leq 1 \ \forall Y \text{ predecessor of } X \} \]

\[ \mathcal{R}_\Lambda = \{ X \in \text{ind}\Lambda : \text{id} Y \leq 1 \ \forall Y \text{ successor of } X \} \]

Definition
An algebra \( \Lambda \) is called laura if \( \mathcal{L}_\Lambda \cup \mathcal{R}_\Lambda \) is cofinite in \( \text{ind}\Lambda \).
\[ \mathcal{L}_\Lambda = \{ X \in \text{ind}\Lambda : \text{pd} \ Y \leq 1 \ \forall \ Y \text{ predecessor of } X \} \]

\[ \mathcal{R}_\Lambda = \{ X \in \text{ind}\Lambda : \text{id} \ Y \leq 1 \ \forall \ Y \text{ successor of } X \} \]

**Definition**

An algebra \( \Lambda \) is called **laura** if \( \mathcal{L}_\Lambda \cup \mathcal{R}_\Lambda \) is cofinite in \( \text{ind} \ \Lambda \)
Example

Let $\Lambda$ be given by the quiver

\[
\begin{array}{ccccccc}
& & & & & & \\
& & & & & & \\
& & & & & & \\
1 & \rightarrow & 2 & \rightarrow & 3 & \leftarrow & 4 & \rightarrow & 5 \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
\end{array}
\]

bound by $\text{rad}^2 \Lambda = 0$. The Auslander-Reiten quiver $\Gamma(\text{mod}\Lambda)$ of $\Lambda$ has a component $\Gamma$ of the following shape
where we identify the two copies of $S_3$, along the vertical dotted lines (note that $\Lambda$ is a laura algebra, having $\Gamma$ as its unique faithful quasi-directed component).

1. $\mathcal{L}_\Lambda = \text{Pred} (S_2)$
2. $\mathcal{R}_\Lambda = \text{Suc} (S_4)$
where we identify the two copies of $S_3$, along the vertical dotted lines (note that $\Lambda$ is a laura algebra, having $\Gamma$ as its unique faithful quasi-directed component).

1. $\mathcal{L}_\Lambda = \text{Pred } (S_2)$
2. $\mathcal{R}_\Lambda = \text{Suc } (S_4)$
A **trisection** of $\text{ind}\Lambda$ is a triple of disjoint full subcategories $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of $\text{ind}\Lambda$ such that:

(a) $\text{ind}\Lambda = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$, and

(b) $\text{Hom}_\Lambda(\mathcal{C}, \mathcal{B}) = \text{Hom}_\Lambda(\mathcal{C}, \mathcal{A}) = \text{Hom}_\Lambda(\mathcal{B}, \mathcal{A}) = 0$.

**Remarks**

1. If any of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ is empty, then the definition above reduces to that of a split torsion pair.

2. If $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is a trisection of $\text{ind}\Lambda$, then $\mathcal{A}$ is closed under predecessors, $\mathcal{C}$ is closed under successors, and $\mathcal{B}$ is *convex* in $\text{ind}\Lambda$. 

**F. U. Coelho**

quasi-directed components
Definition

A trisection of $\text{ind}\Lambda$ is a triple of disjoint full subcategories $(A, B, C)$ of $\text{ind}\Lambda$ such that:

(a) $\text{ind}\Lambda = A \cup B \cup C$, and

(b) $\text{Hom}_\Lambda(C, B) = \text{Hom}_\Lambda(C, A) = \text{Hom}_\Lambda(B, A) = 0$.

Remarks

1. If any of $A, B, C$ is empty, then the definition above reduces to that of a split torsion pair.

2. If $(A, B, C)$ is a trisection of $\text{ind}\Lambda$, then $A$ is closed under predecessors, $C$ is closed under successors, and $B$ is convex in $\text{ind}\Lambda$. 
Trisections

Definition

A trisection of $\text{ind}\Lambda$ is a triple of disjoint full subcategories $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of $\text{ind}\Lambda$ such that:

(a) $\text{ind}\Lambda = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$, and

(b) $\text{Hom}_\Lambda(\mathcal{C}, \mathcal{B}) = \text{Hom}_\Lambda(\mathcal{C}, \mathcal{A}) = \text{Hom}_\Lambda(\mathcal{B}, \mathcal{A}) = 0$.

Remarks

1. If any of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ is empty, then the definition above reduces to that of a split torsion pair.

2. If $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is a trisection of $\text{ind}\Lambda$, then $\mathcal{A}$ is closed under predecessors, $\mathcal{C}$ is closed under successors, and $\mathcal{B}$ is convex in $\text{ind}\Lambda$. 
Definition

A trisection \((\mathcal{A}, \mathcal{B}, \mathcal{C})\) is **separated** provided \(\mathcal{B} \neq \emptyset\) and any morphism \(X \to Y\) with \(X \in \mathcal{A}\) and \(Y \in \mathcal{C}\) factors through \(\text{add}\mathcal{B}\).

Examples

1. If \(\Lambda\) is a representation-infinite hereditary algebra, then \((\mathcal{A}, \mathcal{B}, \mathcal{C})\), where \(\mathcal{A}\) consists of the postprojective modules, \(\mathcal{C}\) of the preinjective, and \(\mathcal{B}\) of the regular, is a separated trisection.

2. If \(\Lambda\) is a quasitilted algebra, then \((\mathcal{L}_\Lambda \setminus \mathcal{R}_\Lambda, \mathcal{L}_\Lambda \cap \mathcal{R}_\Lambda, \mathcal{R}_\Lambda \setminus \mathcal{L}_\Lambda)\) is a separated trisection.
**Definition**

A trisection \((\mathcal{A}, \mathcal{B}, \mathcal{C})\) is separated provided \(\mathcal{B} \neq \emptyset\) and any morphism \(X \to Y\) with \(X \in \mathcal{A}\) and \(Y \in \mathcal{C}\) factors through \(\text{add}\ \mathcal{B}\).

**Examples**

1. If \(\Lambda\) is a representation-infinite hereditary algebra, then \((\mathcal{A}, \mathcal{B}, \mathcal{C})\), where \(\mathcal{A}\) consists of the postprojective modules, \(\mathcal{C}\) of the preinjective, and \(\mathcal{B}\) of the regular, is a separated trisection.

2. If \(\Lambda\) is a quasitilted algebra, then \((\mathcal{L}_\Lambda \setminus \mathcal{R}_\Lambda, \mathcal{L}_\Lambda \cap \mathcal{R}_\Lambda, \mathcal{R}_\Lambda \setminus \mathcal{L}_\Lambda)\) is a separated trisection.
A trisection \((\mathcal{A}, \mathcal{B}, \mathcal{C})\) is separated provided \(\mathcal{B} \neq \emptyset\) and any morphism \(X \to Y\) with \(X \in \mathcal{A}\) and \(Y \in \mathcal{C}\) factors through \(\text{add}\mathcal{B}\).

Examples

1. If \(\Lambda\) is a representation-infinite hereditary algebra, then \((\mathcal{A}, \mathcal{B}, \mathcal{C})\), where \(\mathcal{A}\) consists of the postprojective modules, \(\mathcal{C}\) of the preinjective, and \(\mathcal{B}\) of the regular, is a separated trisection.

2. If \(\Lambda\) is a quasitilted algebra, then \((\mathcal{L}_\Lambda \setminus \mathcal{R}_\Lambda, \mathcal{L}_\Lambda \cap \mathcal{R}_\Lambda, \mathcal{R}_\Lambda \setminus \mathcal{L}_\Lambda)\) is a separated trisection.
Lemma

Let $\mathcal{B}$ be a non-empty finite, connected and convex subcategory of $\text{ind}\Lambda$. Then there is a unique component $\Gamma^\mathcal{B}$ of $\Gamma \mod \Lambda$ such that $\Gamma^\mathcal{B}$ intersects $\mathcal{B}$.

Lemma

Let $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a separated trisection of $\text{ind}\Lambda$, with $\mathcal{B}$ finite and connected, and Let $\Gamma$ be a component of $\Gamma \mod \Lambda$.

(a) If $\Gamma \neq \Gamma^\mathcal{B}$, then either $\Gamma \subset \mathcal{A}$ or $\Gamma \subset \mathcal{C}$.

(b) If $\Gamma \cap \mathcal{A} \neq \emptyset$ and $\Gamma \cap \mathcal{C} \neq \emptyset$, then $\Gamma = \Gamma^\mathcal{B}$. 
**Lemma**

Let $\mathcal{B}$ be a non-empty finite, connected and convex subcategory of $\text{ind}\Lambda$. Then there is a unique component $\Gamma^\mathcal{B}$ of $\Gamma \mod \Lambda$ such that $\Gamma^\mathcal{B}$ intersects $\mathcal{B}$.

**Lemma**

Let $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a separated trisection of $\text{ind}\Lambda$, with $\mathcal{B}$ finite and connected, and Let $\Gamma$ be a component of $\Gamma \mod \Lambda$.

(a) If $\Gamma \neq \Gamma^\mathcal{B}$, then either $\Gamma \subset \mathcal{A}$ or $\Gamma \subset \mathcal{C}$.

(b) If $\Gamma \cap \mathcal{A} \neq \emptyset$ and $\Gamma \cap \mathcal{C} \neq \emptyset$, then $\Gamma = \Gamma^\mathcal{B}$. 
Definition

A trisection \((\mathcal{A}, \mathcal{B}, \mathcal{C})\) is called compact if it is separated and \(\mathcal{B}\) is finite, connected and contains all the projectives and all the injectives in \(\Gamma^\mathcal{B}\).

Remark

An algebra \(\Lambda\) is representation-finite if and only if \((\emptyset, \text{ind}\Lambda, \emptyset)\) is the unique compact trisection of \(\text{ind}\Lambda\).
Definition

A trisection \((\mathcal{A}, \mathcal{B}, \mathcal{C})\) is called \textbf{compact} if it is separated and \(\mathcal{B}\) is finite, connected and contains all the projectives and all the injectives in \(\Gamma^\mathcal{B}\).

Remark

An algebra \(\Lambda\) is representation-finite if and only if \((\emptyset, \text{ind}\Lambda, \emptyset)\) is the unique compact trisection of \(\text{ind}\Lambda\).
The following definition is inspired by the notion of multisection of [RS].

**Definition**

Let $\Gamma$ be a component of $\Gamma(\text{mod}\Lambda)$. A full subcategory $\Delta$ of $\Gamma$ is a core of $\Gamma$ (and $\Gamma$ is said to have a core) if:

(a) $\Delta$ is convex in $\text{ind}\Lambda$.

(b) $\Delta$ intersects each $\tau_\Lambda$-orbit in $\Gamma$, and only finitely many times.

(c) $\Delta$ is almost directed.

**Example**

A complete slice in the connecting component of a tilted algebra is a core in this component.
The following definition is inspired by the notion of multisection of \([RS]\).

**Definition**

Let \(\Gamma\) be a component of \(\Gamma(\text{mod}\Lambda)\). A full subcategory \(\Delta\) of \(\Gamma\) is a **core** of \(\Gamma\) (and \(\Gamma\) is said **to have a core**) if:

- (a) \(\Delta\) is convex in \(\text{ind}\Lambda\).
- (b) \(\Delta\) intersects each \(\tau_{\Lambda}\)-orbit in \(\Gamma\), and only finitely many times.
- (c) \(\Delta\) is almost directed.

**Example**

A complete slice in the connecting component of a tilted algebra is a core in this component.
The following definition is inspired by the notion of multisection of \([RS]\).

**Definition**

Let \(\Gamma\) be a component of \(\Gamma(\text{mod}\Lambda)\). A full subcategory \(\Delta\) of \(\Gamma\) is a core of \(\Gamma\) (and \(\Gamma\) is said to have a core) if:

(a) \(\Delta\) is convex in \(\text{ind}\Lambda\).

(b) \(\Delta\) intersects each \(\tau_\Lambda\)-orbit in \(\Gamma\), and only finitely many times.

(c) \(\Delta\) is almost directed.

**Example**

A complete slice in the connecting component of a tilted algebra is a core in this component.
For the next example of a core we recall the following definition.

**Definition [A]**

Let $\Gamma$ be a component of $\Gamma(\text{mod}\Lambda)$. A full subquiver $\Sigma$ of $\Gamma$ is called a **right** (or **left**) **section** provided:

1. $\Sigma$ is acyclic,
2. $\Sigma$ is convex in $\Gamma$, and
3. for each $Y \in \Gamma$ such that there exists a path from $\Sigma$ to $Y$ (or from $Y$ to $\Sigma$, respectively), there exists a unique $s \geq 0$ (or $s \leq 0$, respectively) such that $\tau_\Lambda^s Y \in \Sigma$. 

F. U. Coelho

quasi-directed components
For the next example of a core we recall the following definition.

**Definition [A]**

Let $\Gamma$ be a component of $\Gamma(\text{mod}\Lambda)$. A full subquiver $\Sigma$ of $\Gamma$ is called a right (or left) section provided:

1. $\Sigma$ is acyclic,
2. $\Sigma$ is convex in $\Gamma$, and
3. for each $Y \in \Gamma$ such that there exists a path from $\Sigma$ to $Y$ (or from $Y$ to $\Sigma$, respectively), there exists a unique $s \geq 0$ (or $s \leq 0$, respectively) such that $\tau_A^s Y \in \Sigma$. 


Example

Let $\Lambda$ be a representation-infinite strict laura algebra, and $\Gamma$ be the non-semiregular component of $\Gamma(\text{mod}\Lambda)$. Let $\Sigma_l$ be a left section, and $\Sigma_r$ be a right section of $\Gamma$. Then the convex hull $\Delta = \mathcal{C}(\Sigma_l \cup \Sigma_r)$ of $\Sigma_l$ and $\Sigma_r$ (that is, the full subcategory consisting of all the modules $M \in \Gamma$ such that there is a path $M' \rightsquigarrow M \rightsquigarrow M''$, with $M', M'' \in \Sigma_l \cup \Sigma_r$) is a core in $\Gamma$, which contains all the non-directed modules of $\Gamma$. 
Main result

Theorem

The following conditions are equivalent for an artin algebra $\Lambda$:

(a) $\Gamma(\text{mod}\Lambda)$ admits a separating quasi-directed component $\Gamma$.
(b) $\text{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.
(c) $\Gamma(\text{mod}\Lambda)$ admits a separating convex component $\Gamma$ having a left section $\Sigma_l$ and a right section $\Sigma_r$ whose convex envelope $\Delta$ is a core in $\Gamma$.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\text{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\text{Hom}_\Lambda(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;
(ii) $\text{Hom}_\Lambda(\Gamma, \Gamma') \neq 0$ implies $\Gamma' \subset \mathcal{C}$.
The following conditions are equivalent for an artin algebra $Λ$:

(a) $Γ(\text{mod}Λ)$ admits a separating quasi-directed component $Γ$.

(b) $\text{ind}Λ$ admits a compact trisection $(A, B, C)$.

(c) $Γ(\text{mod}Λ)$ admits a separating convex component $Γ$ having a left section $Σ_l$ and a right section $Σ_r$ whose convex envelope $Δ$ is a core in $Γ$.

If these conditions are satisfied, and $Γ' ≠ Γ$ is a component of $Γ(\text{mod}Λ)$, then either $Γ' ⊂ A$ or $Γ' ⊂ C$. Moreover,

(i) $\text{Hom}_Λ(Γ', Γ) ≠ 0$ implies $Γ' ⊂ A$;

(ii) $\text{Hom}_Λ(Γ, Γ') ≠ 0$ implies $Γ' ⊂ C$. 

F. U. Coelho
Main result

Theorem

The following conditions are equivalent for an artin algebra $\Lambda$:

(a) $\Gamma(\text{mod}\Lambda)$ admits a separating quasi-directed component $\Gamma$.

(b) $\text{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

(c) $\Gamma(\text{mod}\Lambda)$ admits a separating convex component $\Gamma$ having a left section $\Sigma_l$ and a right section $\Sigma_r$ whose convex envelope $\Delta$ is a core in $\Gamma$.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\text{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\text{Hom}_\Lambda(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;

(ii) $\text{Hom}_\Lambda(\Gamma, \Gamma') \neq 0$ implies $\Gamma' \subset \mathcal{C}$. 

F. U. Coelho quasidirected components
The following conditions are equivalent for an artin algebra $\Lambda$:

(a) $\Gamma(\text{mod}\Lambda)$ admits a separating quasi-directed component $\Gamma$.
(b) $\text{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.
(c) $\Gamma(\text{mod}\Lambda)$ admits a separating convex component $\Gamma$ having a left section $\Sigma_l$ and a right section $\Sigma_r$ whose convex envelope $\Delta$ is a core in $\Gamma$.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\text{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\text{Hom}_\Lambda(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;
(ii) $\text{Hom}_\Lambda(\Gamma, \Gamma') \neq 0$ implies $\Gamma' \subset \mathcal{C}$. 
Main result

Theorem

The following conditions are equivalent for an artin algebra $\Lambda$:

(a) $\Gamma(\text{mod}\Lambda)$ admits a separating quasi-directed component $\Gamma$.

(b) $\text{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

(c) $\Gamma(\text{mod}\Lambda)$ admits a separating convex component $\Gamma$ having a left section $\Sigma_l$ and a right section $\Sigma_r$ whose convex envelope $\Delta$ is a core in $\Gamma$.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\text{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\text{Hom}_{\Lambda}(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;

(ii) $\text{Hom}_{\Lambda}(\Gamma, \Gamma') \neq 0$ implies $\Gamma' \subset \mathcal{C}$.
Main result

Theorem

The following conditions are equivalent for an artin algebra $\Lambda$:

(a) $\Gamma(\text{mod}\Lambda)$ admits a separating quasi-directed component $\Gamma$.

(b) $\text{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

(c) $\Gamma(\text{mod}\Lambda)$ admits a separating convex component $\Gamma$ having a left section $\Sigma_l$ and a right section $\Sigma_r$ whose convex envelope $\Delta$ is a core in $\Gamma$.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\text{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\text{Hom}_\Lambda(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;

(ii) $\text{Hom}_\Lambda(\Gamma, \Gamma') \neq 0$ implies $\Gamma' \subset \mathcal{C}$. 
Consequences

Let $\Lambda$ be an algebra admitting a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

### Description of $\Gamma(\text{mod } \Lambda)$

1. the separating quasi-directed convex component $\Gamma^B$
2. components lying in $\mathcal{A}$ are components of its support algebra $A = \text{End}(\bigoplus_{P_x \in \mathcal{A}} P_x)$
3. components lying in $\mathcal{C}$ are components of the support algebra of $\mathcal{C}$

1. either $\Gamma^B$ is non-semiregular, in which case the algebra $\Lambda/\text{Ann } \Gamma^B$ is a laura algebra;
2. or $\Gamma^B$ is the connecting component of the algebra $\Lambda/\text{Ann } \Gamma^B$ which is tilted.
Let $\Lambda$ be an algebra admitting a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

**Description of $\Gamma(\text{mod } \Lambda)$**

1. the separating quasi-directed convex component $\Gamma^B$
2. components lying in $\mathcal{A}$ are components of its support algebra $A = \text{End}(\bigoplus_{P_x \in \mathcal{A}} P_x)$
3. components lying in $\mathcal{C}$ are components of the support algebra of $\mathcal{C}$

1. either $\Gamma^B$ is non-semiregular, in which case the algebra $\Lambda/\text{Ann } \Gamma^B$ is a laura algebra;
2. or $\Gamma^B$ is the connecting component of the algebra $\Lambda/\text{Ann } \Gamma^B$ which is tilted.


Thank You!