From trisections in module categories to quasi-directed components

Flávio Ulhoa Coelho - IME-USP

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E. ALVARES, I ASSEM, F. U. COELHO, M. I. PEÑA, S. TREPODE, *From trisections in module categories to quasi-directed ccomponents*, J. Alg. Appl. **10**,3 (2011) 409-433.

- Λ - basic and connected artin algebra
- mod Λ - category of finitely generated right Λ-modules
- ind Λ - full subcategory of mod Λ consisting of exactly one representative from each isomorphism class of indecomposable Λ -modules.
- Γ (mod Λ) - the Auslander-Reiten quiver of Λ.

Preliminaries

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Definition

A generalised standard component $\Gamma \subset \Gamma(\text{ mod } \Lambda)$ is quasi-directed provided there exist at most finitely many modules lying in oriented cycles.

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laura algebras

$$\mathcal{L}_\Lambda = \{X \in \text{ ind}\Lambda : \text{ pd}\, Y \leq 1 \ \ \forall Y \text{ predecessor of } X\}$$

$$\mathcal{R}_\Lambda = \{X \in \text{ ind}\Lambda : \text{ id}\, Y \leq 1 \ \ \forall Y \text{ successor of } X\}$$

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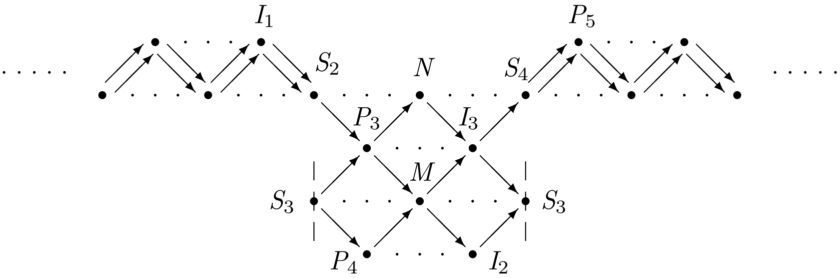
Let Λ be given by the quiver

$$\begin{tikzcd} \underset{1}{\bullet} & \underset{2}{\bullet} \arrow[l, shift left] \arrow[l, shift right] & \underset{3}{\bullet} \arrow[l] \arrow[loop above] & \underset{4}{\bullet} \arrow[l] & \underset{5}{\bullet} \arrow[l, shift left] \arrow[l, shift right] \end{tikzcd}$$

bound by $\mathrm{rad}^2\Lambda = 0$. The Auslander-Reiten quiver $\Gamma(\mathrm{mod}\Lambda)$ of Λ has a component Γ of the following shape

Example



where we identify the two copies of S_3, along the vertical dotted lines (note that Λ is a laura algebra, having Γ as its unique faithful quasi-directed component).

1. $\mathcal{L}_\Lambda = \mathrm{Pred}\ (S_2)$
2. $\mathcal{R}_\Lambda = \mathrm{Suc}\ (S_4)$

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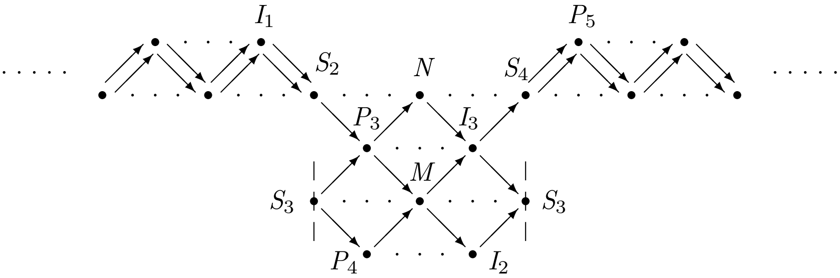

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Trisections

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A trisection of $\mathrm{ind}\Lambda$ is a triple of disjoint full subcategories $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of $\mathrm{ind}\Lambda$ such that:

(a) $\mathrm{ind}\Lambda = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$, and

(b) $\mathrm{Hom}_\Lambda(\mathcal{C}, \mathcal{B}) = \mathrm{Hom}_\Lambda(\mathcal{C}, \mathcal{A}) = \mathrm{Hom}_\Lambda(\mathcal{B}, \mathcal{A}) = 0$.

Remarks

1. If any of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ is empty, then the definition above reduces to that of a split torsion pair.
2. If $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is a trisection of $\mathrm{ind}\Lambda$, then $\mathcal{A}$ is closed under predecessors, $\mathcal{C}$ is closed under successors, and $\mathcal{B}$ is *convex* in $\mathrm{ind}\Lambda$.

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Separated trisections

Definition

A trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is separated provided $\mathcal{B} \neq \emptyset$ and any morphism $X \to Y$ with $X \in \mathcal{A}$ and $Y \in \mathcal{C}$ factors through $\mathrm{add}\mathcal{B}$.

Examples

1. If Λ is a representation-infinite hereditary algebra, then $(\mathcal{A}, \mathcal{B}, \mathcal{C})$, where $\mathcal{A}$ consists of the postprojective modules, $\mathcal{C}$ of the preinjective, and $\mathcal{B}$ of the regular, is a separated trisection.
2. If Λ is a quasitilted algebra, then $(\mathcal{L}_\Lambda \setminus \mathcal{R}_\Lambda, \mathcal{L}_\Lambda \cap \mathcal{R}_\Lambda, \mathcal{R}_\Lambda \setminus \mathcal{L}_\Lambda)$ is a separated trisection.

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Lemma

Let $\mathcal{B}$ be a non-empty finite, connected and convex subcategory of $\mathrm{ind}\Lambda$. Then there is a unique component $\Gamma^{\mathcal{B}}$ of Γ (mod Λ) such that $\Gamma^{\mathcal{B}}$ intersects $\mathcal{B}$.

Lemma

Let $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ be a separated trisection of $\mathrm{ind}\Lambda$, with $\mathcal{B}$ finite and connected, and Let Γ be a component of $\Gamma(\mathrm{mod}\Lambda)$.

(a) If $\Gamma \neq \Gamma^{\mathcal{B}}$, then either $\Gamma \subset \mathcal{A}$ or $\Gamma \subset \mathcal{C}$.

(b) If $\Gamma \cap \mathcal{A} \neq \emptyset$ and $\Gamma \cap \mathcal{C} \neq \emptyset$, then $\Gamma = \Gamma^{\mathcal{B}}$.

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A trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is called compact if it is separated and $\mathcal{B}$ is finite, connected and contains all the projectives and all the injectives in $\Gamma^{\mathcal{B}}$.

Remark

An algebra Λ is representation-finite if and only if $(\emptyset, \mathrm{ind}\Lambda, \emptyset)$ is the unique compact trisection of $\mathrm{ind}\Lambda$.

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Core

The following definition is inspired by the notion of multisection of [**RS**].

Definition

Let Γ be a component of $\Gamma(\mathrm{mod}\Lambda)$. A full subcategory Δ of Γ is a core of Γ (and Γ is said to have a core) if:

(a) Δ is convex in $\mathrm{ind}\Lambda$.

(b) Δ intersects each τ_Λ-orbit in Γ, and only finitely many times.

(c) Δ is almost directed.

Example

A complete slice in the connecting component of a tilted algebra is a core in this component

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For the next example of a core we recall the following definition.

Definition [A]

Let Γ be a component of $\Gamma(\text{mod}\Lambda)$. A full subquiver Σ of Γ is called a right (or left) section provided:

(1) Σ is acyclic,

(2) Σ is convex in Γ, and

(3) for each $Y \in \Gamma$ such that there exists a path from Σ to Y (or from Y to Σ, respectively), there exists a unique $s \geq 0$ (or $s \leq 0$, respectively) such that $\tau_\Lambda^s Y \in \Sigma$.

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Example

Let Λ be a representation-infinite strict laura algebra, and Γ be the non-semiregular component of $\Gamma(\mathrm{mod}\Lambda)$. Let Σ_l be a left section, and Σ_r be a right section of Γ. Then the convex hull $\Delta = \mathcal{C}(\Sigma_l \cup \Sigma_r)$ of Σ_l and Σ_r (that is, the full subcategory consisting of all the modules $M \in \Gamma$ such that there is a path $M' \rightsquigarrow M \rightsquigarrow M''$, with $M', M'' \in \Sigma_l \cup \Sigma_r$) is a core in Γ, which contains all the non-directed modules of Γ.

Main result

Theorem

The following conditions are equivalent for an artin algebra Λ:

(a) $\Gamma(\mathrm{mod}\Lambda)$ admits a separating quasi-directed component Γ.

(b) $\mathrm{ind}\Lambda$ admits a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

(c) $\Gamma(\mathrm{mod}\Lambda)$ admits a separating convex component Γ having a left section Σ_l and a right section Σ_r whose convex envelope Δ is a core in Γ.

If these conditions are satisfied, and $\Gamma' \neq \Gamma$ is a component of $\Gamma(\mathrm{mod}\Lambda)$, then either $\Gamma' \subset \mathcal{A}$ or $\Gamma' \subset \mathcal{C}$. Moreover,

(i) $\mathrm{Hom}_\Lambda(\Gamma', \Gamma) \neq 0$ implies $\Gamma' \subset \mathcal{A}$;

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Consequences

Let Λ be an algebra admitting a compact trisection $(\mathcal{A}, \mathcal{B}, \mathcal{C})$.

Description of $\Gamma(\mathrm{mod}\ \Lambda)$

1. the separating quasi-directed convex component $\Gamma^{\mathcal{B}}$
2. components lying in $\mathcal{A}$ are components of its support algebra $A = \mathrm{End}(\bigoplus_{P_x \in \mathcal{A}} P_x)$
3. components lying in $\mathcal{C}$ are components of the support algebra of $\mathcal{C}$

1. either $\Gamma^{\mathcal{B}}$ is non-semiregular, in which case the algebra $\Lambda / Ann\ \Gamma^{\mathcal{B}}$ is a laura algebra;
2. or $\Gamma^{\mathcal{B}}$ is the connecting component of the algebra $\Lambda / Ann\ \Gamma^{\mathcal{B}}$ which is tilted.

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References

- [**A**] I. Assem, *Left sections and the left part of an Artin algebra*, Colloq. Math. **116**(2) (2009) 273-300.
- [**AC**] I. Assem, I.,F. U. Coelho, *Two-sided gluings of tilted algebras*, J. Algebra **269** (2) (2003) 456-479.
- [**RS**] I. Reiten, A. Skowroński, *Generalized double tilted algebras*, J. Math. Soc. Japan **56**(1) (2004) 269-288.

Thank You !