Weyl modules for generalized current algebras

Ghislain Fourier

Universität zu Köln

12.08.2012 / ICRA 2012 - Bielefeld

We will fix in this talk the following

- $\mathfrak{g}$, a simple complex Lie algebra.
- A, a commutative, associative algebra (over $\mathbb{C}$).

The main object of this talk is the Lie algebra

$$\mathfrak{g} \otimes A$$

with bracket

$$[x \otimes a, y \otimes b] := [x, y] \otimes ab.$$

This might be seen, in certain cases, as the Lie algebra of regular maps on max A with values in $\mathfrak{g}$.

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We are interested in locally finite-dimensional modules for $\mathfrak{g} \otimes A$, induced by a one-dimensional module (copying the construction of simple finite-dimensional modules for $\mathfrak{g}$).

For this

- Let $\mathbb{C}_\lambda$ a one-dimensional module for $\mathfrak{b}$.
- To obtain a locally finite-dimensional module: $\lambda \in P^+$.
- Extend to an action of $\mathfrak{b} \otimes A$.
- Weights should be bounded, so $\mathfrak{n}^+ \otimes A$ acts by 0.

Denote this module by M_λ.

In fact, M_λ is isomorphic to $\mathbf{U}(\mathfrak{b} \otimes A)/(\mathfrak{b} \otimes A, h - \lambda(h))$.

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The induced module

$$\operatorname{Ind}_{\mathbf{U}(\mathfrak{b}\otimes A)}^{\mathbf{U}(\mathfrak{g}\otimes A)} M_\lambda$$

might be seen as an analog of a Verma module.

We consider the maximal locally finite–dimensional (for the $\mathfrak{g}$-action) quotient of this module.

- This quotient exists.
- This quotient is non-trivial.

Denote this quotient by

$$W_A(\lambda).$$

It is called the *(global) Weyl module*.

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Integrability $\Rightarrow$ certain relations ("Garland equation")
$\Rightarrow$ $(W_A(\lambda))_\lambda$ is proper quotient of $\mathrm{Ind}_{\mathbf{U}(\mathfrak{b}\otimes A)}^{\mathbf{U}(\mathfrak{g}\otimes A)} M_\lambda$.

By construction, this weight space is a commutative algebra (as a quotient of $\mathbf{U}(\mathfrak{h} \otimes A)$), denoted by $\mathbf{A}_\lambda$.

$W_A(\lambda)$ admits a right $\mathbf{A}_\lambda$-module structure, hence it is a $(\mathfrak{g} \otimes A, \mathbf{A}_\lambda)$-bimodule.

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We will consider certain choices of A and see what is known about $W_A(\lambda)$.

First case: $A = \mathbb{C}$.
Then $W_A(\lambda) = L(\lambda)$, the simple finite-dimensional $\mathfrak{g}$-module of highest weight λ (known for a long time).

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Second case: A finitely generated.

Then

Theorem

For all $\lambda \in P^+$

- $\mathbf{A}_\lambda$ *is a finitely generated algebra.*
- $W(\lambda)$ *finitely generated as an* $\mathbf{A}_\lambda$*-module.*

Even more

Lemma

If $J(A) = 0$ then

$$\mathbf{A}_\lambda = S^{\lambda_1} A \otimes \ldots \otimes S^{\lambda_n} A.$$

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Third case: $A = \mathbb{C}[t]$.

Intensively studied in the past two decades...
Motivation by Weyl modules for the quantum affine algebra (Chari-Pressley). They conjectured

Conjecture

$W_A(\lambda)$ is a free right $\mathbf{A}_\lambda$-module of finite rank. The rank is the product of the ranks of $W_A(\omega_i)$.

This is now a theorem by works of Chari, Pressley, Loktev, Littelmann, Naoi to name a few.
It is proven by passing to *local Weyl modules*, that is

$$W_A(\lambda) \otimes_{\mathbf{A}_\lambda} M$$

for a one-dimensional left $\mathbf{A}_\lambda$-module (Note that $\mathbf{A}_\lambda$ is a polynomial ring in this case).

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Fourth case: max A smooth

$W_A(\omega_i)$ is a free $\mathbf{A}_\lambda$-module, tha rank is obtained from the dimension of max A.

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Fifth case: $A = \mathbb{C}[t]/(t^N)$.

In this case, $\mathbf{A}_\lambda$ has a unique maximal ideal, this implies that there is a unique simple $\mathfrak{g} \otimes A$ module of highest weight λ (the evaluation module).

Question: What is the unique local Weyl module? Can one give a dimension/character formula as in the case $\mathbb{C}[t]$? What is a minimal set of generators for $W_A(\lambda)$?

Answer is still open, but

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