2-hereditary algebras and quivers with potential

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References

This talk is based on joint work with Osamu Iyama and Steffen Oppermann. In progress.

- M. Herschend and O. Iyama, *Selfinjective quivers with potential and 2-representation-finite algebras*, Compos. Math. **147** (2011), no. 6, 1885–1920.
- M. Herschend, O. Iyama and S. Oppermann, *n-representation infinite algebras*, Preprint, (2012), arXiv:1205.1272.

2-hereditary algebras

K : algebrically closed field, $D := \mathrm{Hom}_K(-, K)$.
Λ : basic, ring indecomposable, finite dimensional K-algebra.
$\mathrm{gl.dim}\,\Lambda = 2$
$\nu_2 := - \overset{\mathbf{L}}{\otimes} D\Lambda \circ [-2] \colon \mathrm{D^b}(\mathrm{mod}\,\Lambda) \to \mathrm{D^b}(\mathrm{mod}\,\Lambda)$

- Λ : 2-hereditary $\iff$ $\nu_2^i \Lambda \in \mathrm{D}^{2\mathbb{Z}}(\mathrm{mod}\,\Lambda)$ for all $i \in \mathbb{Z}$.
- Λ : 2-representation finite (2-RF)
  $\iff$ $\forall P$ indec. proj. $\nu_2^{-m_P} P$ is injective for some $m_P \geq 0$.
- Λ : 2-representation infinite (2-RI)
  $\iff$ $\nu_2^{-i} \Lambda \in \mathrm{mod}\,\Lambda$ for all $i \geq 0$.

Theorem

Λ is 2-hereditary $\iff$ Λ is 2-RF or Λ is 2-RI.

Preprojective algebras

$$\Pi(\Lambda) := T_\Lambda(\mathrm{Ext}^2_\Lambda(D\Lambda,\Lambda)) \simeq \bigoplus_{i\geq 0} \mathrm{Hom}_{\mathrm{D}}(\Lambda, \nu_2^{-i}\Lambda) \simeq \bigoplus_{i\geq 0} \tau_2^{-i}\Lambda$$

Theorem

Λ : 2-*RF* $\iff$ $\Pi(\Lambda)$: *finite dimensional selfinjective.*

Λ : 2-*RI* $\iff$ $\Pi(\Lambda)$: *bimodule* 3-*CY of Gorenstein parameter* 1*.*

Remark

- 2-*RF case by Iyama-Oppermann.*
- 2-*RI case by Amiot-Iyama-Reiten, Keller. See also Minamoto-Mori.*
- $\Lambda = \Pi(\Lambda)_0$.
- $\Pi(\Lambda)$ *can be described using quivers with potential.*

Quivers with potential and cuts

(Q, W, C) : quiver with potential and cut

- Q : quiver
- W : potential (linear combination of cycles in Q)
- C : cut ($C \subset Q_1$, W homogenous of degree 1)

$\mathcal{P}(Q, W, C) := KQ/\langle \partial_a W \mid a \in Q_1 \rangle$ graded by C.

$\mathcal{P}(Q, W)_C := \mathcal{P}(Q, W, C)_0 = KQ_C/\langle \partial_c W \mid c \in C \rangle$

We further assume that $\langle \partial_c W \mid c \in C \rangle < KQ_C$ is admissible.

Construction of preprojective algebras

For $\Lambda = KQ/\langle r_1, \ldots, r_m \rangle$, we define (Q', W, C) such that $\mathcal{P}(Q, W, C) \simeq \Pi(\Lambda)$.

- $Q'_0 = Q_0$
- $Q'_1 = Q_1 \coprod C$
- $C = \{\rho_i : e(r_i) \to s(r_i) \mid 1 \leq i \leq m\}$
- $W = \sum_{i=1}^{n} \rho_i r_i$

By Keller $\mathcal{P}(Q, W, C) \simeq \Pi(\Lambda)$.

$$\mathcal{P}(Q', W)_C \simeq KQ/\langle \partial_{\rho_i} W \mid 1 \leq i \leq m \rangle = \Lambda$$

Structure theorem for 2-hereditary algebras

Theorem

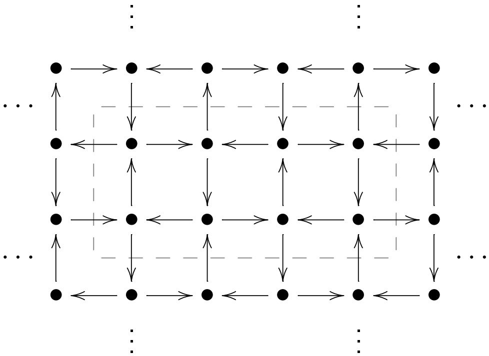
Let (Q, W, C) be a quiver with potential and cut.

- *If (Q, W) is selfinjective, then $\mathcal{P}(Q, W)_C$ is 2-RF.*
- *If (Q, W) is 3-CY, then $\mathcal{P}(Q, W)_C$ is 2-RI.*
- *Every 2-hereditary algebra appears in this way.*

Sources of 3-CY quivers with potential:

- Skew polynomial algebras in three variables.
- Consistent dimer models.

Example 1



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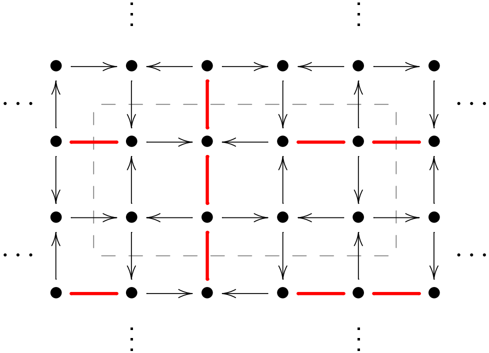

P : simple projective. $T := (\Lambda/P) \oplus \tau_2^-(P)$.
$\mathrm{End}_\Lambda(T)$: 2-APR tilt of Λ.

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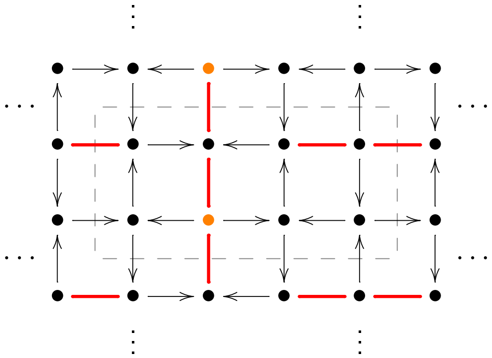

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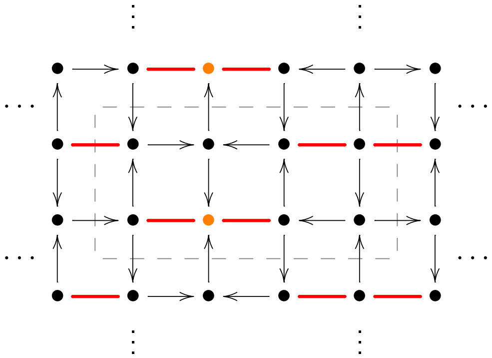

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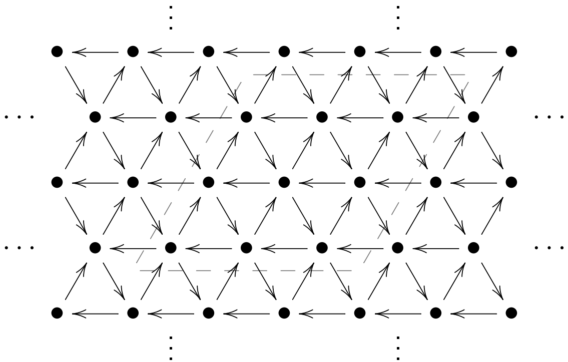

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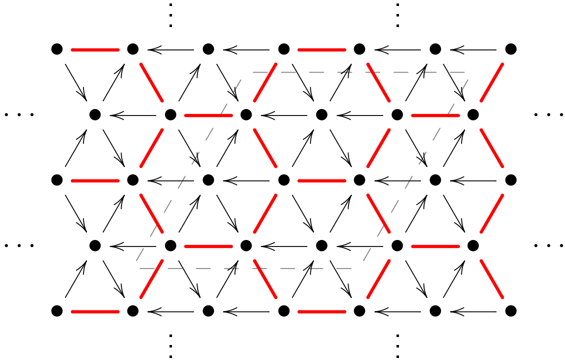

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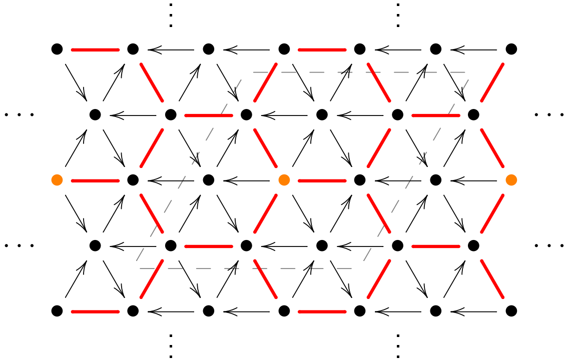

Example 2