On selfinjective algebras without short cycles in the component quiver

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ICRA 2012, Bielefeld, 14 August 2012

Plan of the talk

1. Set-up
   - Basics
   - Component quiver and its properties
   - Almost concealed canonical algebras
   - Tubular algebras
   - Selfinjective orbit algebras
2. Main result
3. Idea of the proof

- By an algebra we mean a basic, connected artin algebra over a commutative artin ring k.
- By mod A we denote the category of finitely generated right A-modules.
- Γ_A denotes AR-quiver of an algebra A.
- A component $\mathcal{C}$ in Γ_A is called generalized standard if for any $X, Y \in \mathcal{C}$ we have $\mathrm{rad}_A^{\infty}(X, Y) = 0$.

[Skowroński]: Let $\mathcal{C}$ be a generalized standard component of Γ_A. Then $\mathcal{C}$ is almost periodic (all but finitely many τ_A-orbits are periodic).

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Component quiver

The component quiver Σ_A of an algebra A is a quiver whose:

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components of the AR-quiver Γ_A

arrows

$\mathcal{C} \longrightarrow \mathcal{D}$ in Σ_A, where $\mathcal{C}, \mathcal{D} \in \Gamma_A$

$\Updownarrow$

$\mathrm{rad}_A^{\infty}(X, Y) \neq 0$ for some modules X in $\mathcal{C}$ and Y in $\mathcal{D}$

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- Σ_A has no loop $\Longrightarrow$ every component in Γ_A is generalized standard.
- Σ_A is acyclic $\Longleftrightarrow$ A is generically of polynomial growth.
- Σ_A has no short cycle $\Longrightarrow$ modules in acyclic components are uniquely determined by their images in $K_0(A)$.

[Jaworska-Malicki-Skowroński]: Let $\mathcal{C}$ and $\mathcal{D}$ be components of Γ_A. Assume $\mathcal{C}$ is not a stable tube of rank 1 and does not lie on short cycle in Σ_A.

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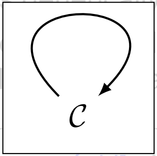

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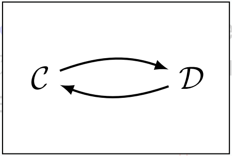

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$[\mathcal{D}] = \{[X] \in K_0(A) \mid X \in \mathcal{D}\}$

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Almost concealed canonical algebras

- Let Λ be a canonical algebra in the sens of Ringel.
- Then Γ_Λ admits a canonical decomposition

$$\Gamma_\Lambda = \mathcal{P}^\Lambda \vee \mathcal{T}^\Lambda \vee \mathcal{Q}^\Lambda$$

with $\mathcal{T}^\Lambda$ the canonical family of stable tubes separating $\mathcal{P}^\Lambda$ from $\mathcal{Q}^\Lambda$.

- An algebra B is said to be an almost concealed canonical algebra if B is the endomorphism algebra $\mathrm{End}_\Lambda(T)$ of a tilting module T from the additive category $\mathrm{add}(\mathcal{P}^\Lambda \vee \mathcal{T}^\Lambda)$.

Moreover, Γ_B admits the canonical decomposition $\Gamma_B = \mathcal{P}^B \vee \mathcal{T}^B \vee \mathcal{Q}^B$ with $\mathcal{T}^B$ the family of ray tubes (i.e. components obtained from stable tubes by ray insertions).

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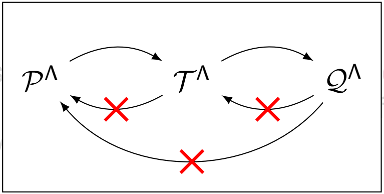

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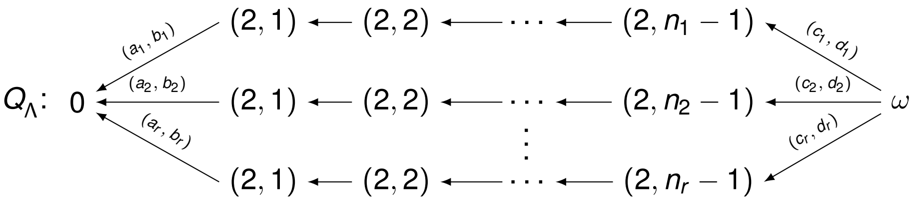
Q_Λ: 0
(a_1, b_1)
(a_2, b_2)
(a_r, b_r)
(2, 1) (2, 2) ⋯ $(2, n_1 - 1)$
(2, 1) (2, 2) ⋯ $(2, n_2 - 1)$
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(c_1, d_1)
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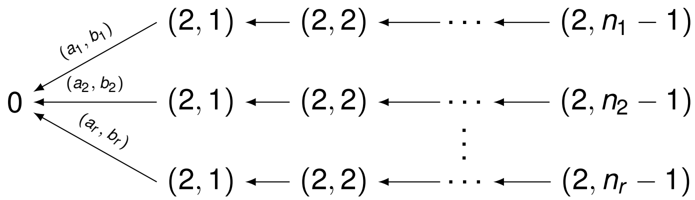
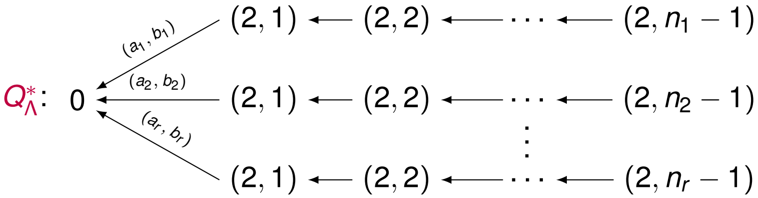
ω

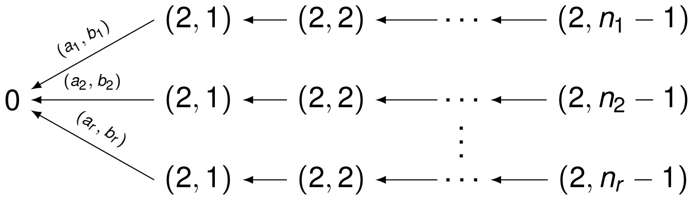
Q^*_Λ:





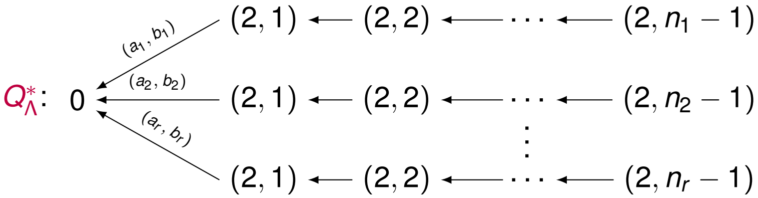
We say that a canonical algebra Λ is of

Q_Λ^*:



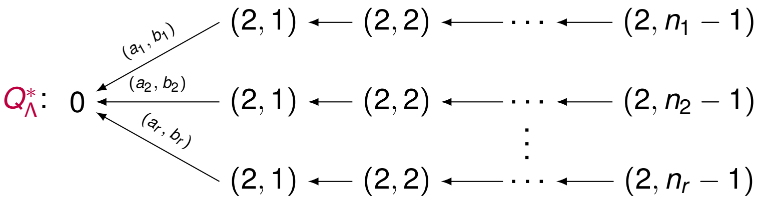
We say that a canonical algebra Λ is of

> Euclidean type if Q_Λ^* is a Dynkin quiver.



We say that a canonical algebra Λ is of

tubular type if Q_Λ^* is a Euclidean quiver.



We say that a canonical algebra Λ is of

wild type in the remaining cases.

Tubular algebras

An almost concealed canonical algebra of type Λ, where Λ is a canonical algebra of tubular type, is called a tubular algebra.

For a field k of characteristic different from 2, the exceptional tubular algebra B_{ex} is given by the following ordinary quiver

and the relations $\alpha\zeta = \gamma\eta$, $\alpha\mu = \gamma\omega$, $\sigma\zeta = \beta\eta$ and $\sigma\mu = -\beta\omega$.

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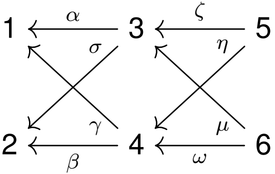

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An automorphism φ of the exceptional tubular algebra B_{ex} is said to be distinguished if:

$$\varphi(\gamma) = a\sigma,\ \varphi(\sigma) = b\gamma,\ \varphi(\beta) = c\alpha,\ \varphi(\alpha) = d\beta,\ \varphi(\mu) = e\eta,\ \varphi(\eta) = r\mu,$$
$$\varphi(\omega) = u\zeta \text{ and } \varphi(\zeta) = v\omega$$

for $a, b, c, d, e, r, u, v \in k \setminus \{0\}$ satisfying the following relations

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Selfinjective orbit algebras

- For an algebra A, we denote by D the standard duality $\mathrm{Hom}_k(-, E)$ on mod A, where E is a minimal injective cogenerator in mod k.
- An algebra A is selfinjective if and only if $A \cong D(A)$ in mod A.
- A component $\mathcal{C}$ in Γ_A, where A is selfinjective algebra, is called a quasitube if its stable part $\mathcal{C}^s$ is a stable tube.

A = selfinjective algebra, $\mathcal{C}$ = a generalized standard component of Γ_A

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$\mathcal{C}$ is a quasitube or $\mathcal{C}^s = \mathbb{Z}\Delta$ for a finite acyclic valued quiver Δ.

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The repetitive algebra $\widehat{B}$ of B is an algebra (without identity) whose k-module structure is that of

$$\bigoplus_{m\in\mathbb{Z}}(B_m \oplus \mathrm{D}(B)_m)$$

where $B_m = B$ and $\mathrm{D}(B)_m = \mathrm{D}(B)$ for all $m \in \mathbb{Z}$, and the multiplication is defined by

$$(a_m, f_m)_m \cdot (b_m, g_m)_m = (a_m b_m, a_m g_m + f_m b_{m-1})_m$$

for $a_m, b_m \in B_m$, $f_m, g_m \in \mathrm{D}(B)_m$.

The repetitive algebra $\widehat{B}$ of B is an algebra (without identity) whose k-module structure is that of

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$\mathcal{E} = \{e_i \mid 1 \leq i \leq n\}$ a fixed set of orthogonal primitive idempotents of B with $1_B = e_1 + \cdots + e_n$

$\widehat{\mathcal{E}} = \{e_{m,i} \mid m \in \mathbb{Z},\ 1 \leq i \leq n\}$ a corresponding set of orthogonal primitive idempotents of $\widehat{B}$ such that

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- By an automorphism of $\widehat{B}$ we mean a k-algebra automorphism of $\widehat{B}$ which fixes the chosen set $\widehat{\mathcal{E}}$ of orthogonal primitive idempotents of $\widehat{B}$.
- A group G of automorphisms of $\widehat{B}$ is said to be admissible if the induced action of G on $\widehat{\mathcal{E}}$ is *free and has finitely many orbits*.
- The orbit algebra $\widehat{B}/G$ is *a finite dimensional selfinjective algebra* and the G-orbits in $\widehat{\mathcal{E}}$ form a canonical set of orthogonal primitive idempotents of $\widehat{B}/G$ whose sum is the identity of $\widehat{B}/G$.

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We denote by $\nu_{\widehat{B}}$ the Nakayama automorphism of $\widehat{B}$ such that

$$\nu_{\widehat{B}}(e_{m,i}) = e_{m+1,i}$$

for all $m \in \mathbb{Z}$, $1 \leq i \leq n$.

An automorphism φ of $\widehat{B}$ is said to be

1. positive if $\varphi(B_m) \subseteq \sum_{j \geq m} B_j$ for any $m \in \mathbb{Z}$.
2. rigid if $\varphi(B_m) = B_m$ for any $m \in \mathbb{Z}$.
3. strictly positive if φ is positive but not rigid.

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Main result

[K.'2012]: Let A be a basic, connected, selfinjective artin algebra of infinite representation type, over a commutative artin ring k. TFAE:

(i) The component quiver Σ_A has no short cycles.

(ii) k is a field and A is isomorphic to an orbit algebra $\widehat{B}/G$, where B is a tilted algebra of Euclidean type or a tubular algebra over k and G is an infinite cyclic group of automorphisms of $\widehat{B}$ of one of the following forms:

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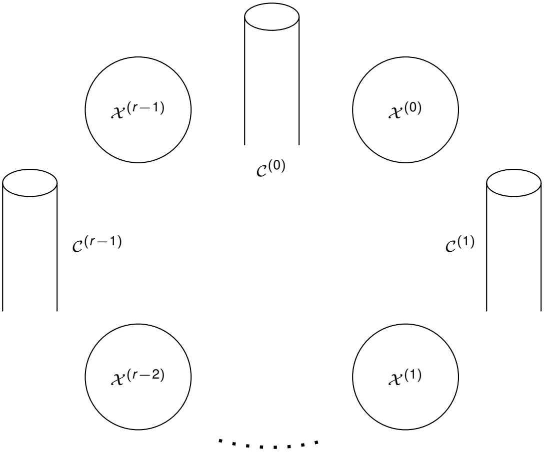
The structure of Γ_A



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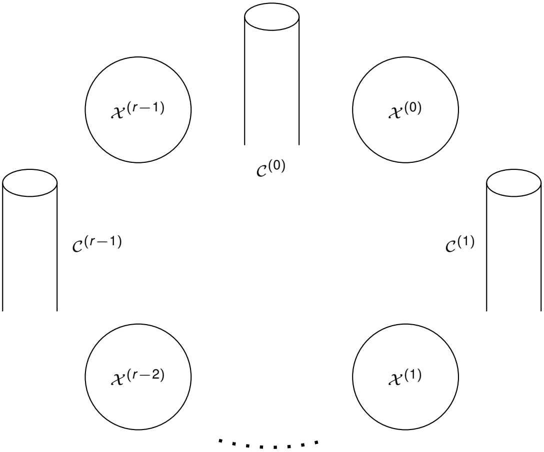

for some integer $r \geq 1$, where $A = \widehat{B}/G$ and each $\mathcal{C}^{(i)}$ is an **infinite family of quasitubes**

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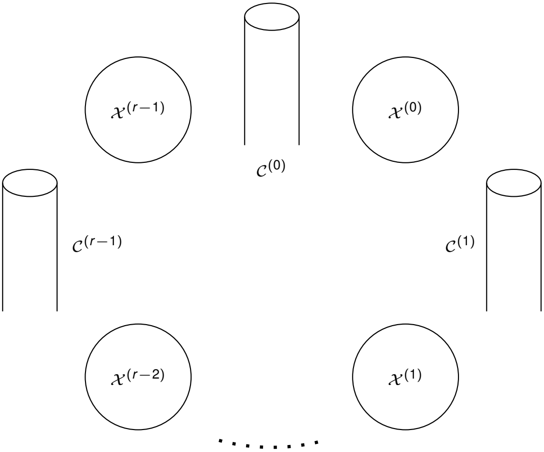

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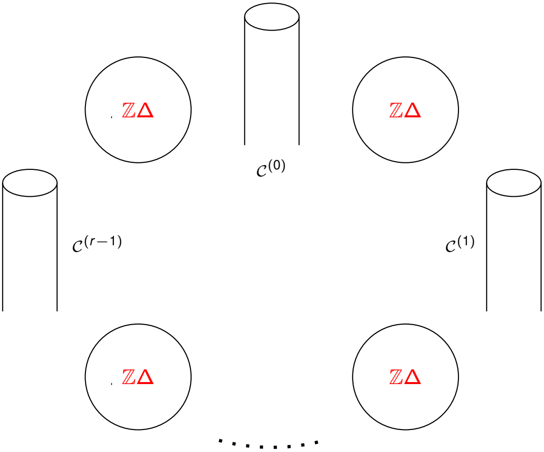

B is tilted algebra of Euclidean type

$\Downarrow$

every $\mathcal{X}^{(i)}$ is an acyclic component of Euclidean type (the stable part is of the form $\mathbb{Z}\Delta$ for a Euclidean quiver Δ)

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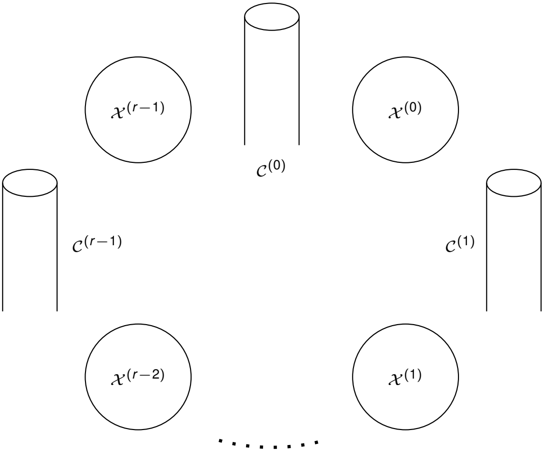

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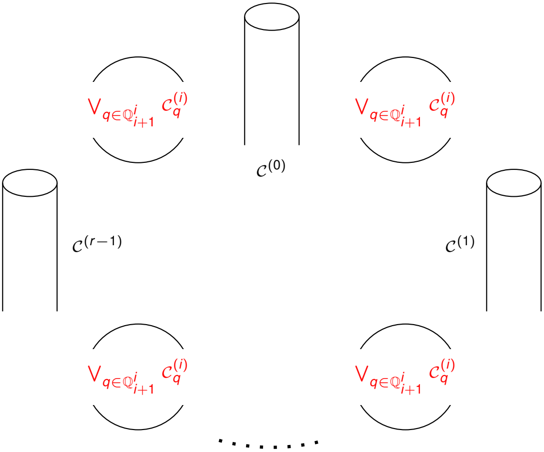

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$\mathcal{X}^{(i)} = \bigvee_{q\in\mathbb{Q}^i_{i+1}} \mathcal{C}^{(i)}_q$, where $\mathcal{C}^{(i)}_q$ is an infinite family of stable tubes for each $q \in \mathbb{Q}^i_{i+1} = \mathbb{Q} \cap (i, i+1)$

for some integer $r \geq 1$, where $A = \widehat{B}/G$ and each $\mathcal{C}^{(i)}$ is an **infinite family of quasitubes**

First step

[K.-Skowroński-Yamagata]: Let A be a basic, connected, selfinjective artin algebra. **TFAE**:

(i) Γ_A admits a family $\mathcal{C} = (\mathcal{C}_i)_{i \in I}$ of quasitubes having common composition factors, closed on composition factors, and consisting of modules which do not lie on infinite short cycles in mod A.

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> X - indecomposable A-module
> If $[X] = [Y]$ for some $Y \in \mathcal{C}$ then $X \in \mathcal{C}$.

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> $X \xrightarrow{f} Y \xrightarrow{g} X$
> g or f is from rad_A^{∞}

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(c) $G = (\varphi\nu_{\widehat{B}}^{2})$, for B of Euclidean or wild type and φ a rigid automorphism of $\widehat{B}$ acting freely on the nonstable tubes of the unique separating family $\mathcal{T}^B$ of ray tubes of Γ_B,

where $\nu_{\widehat{B}}$ is the Nakayama automorphism of $\widehat{B}$.

Second step

(1) In the second step we show that tubular algebras except the exceptional one have fixed points.

(2) Next, we show that automorphism $\varphi : \widehat{B} \to \widehat{B}$, from the statement of the theorem, is of the desired form.

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For Further Reading

M. Karpicz
On selfinjective algebras without short cycles in the component quiver.
Preprint, Toruń 2012.

M. Karpicz, A. Skowroński, K. Yamagata
On selfinjective artin algebras having generalized standard quasitubes.
J. Pure Appl. Algebra 215 (2011) 2738–2760