Module varieties with dense orbits in every component

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Always $K = \overline{K}$ and $A = KQ/I$ finite dimensional algebra

- mod$(A, \underline{d})$: module variety
(matrix representations of A of dim vector $\underline{d}$)

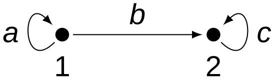
- Def: Say A has Dense Orbit property (DO) if:
for all $\underline{d}$, every irreducible component of mod$(A, \underline{d})$ has a dense orbit.

- Def (again): Say A has Dense Orbit property (DO) if: for all $\underline{d}$, every irreducible component of $\mathrm{mod}(A, \underline{d})$ has a dense orbit.
- If A is finite rep type then A is DO.
- Observation: If $I = 0$ (hereditary algebras)

$$A \text{ is DO} \iff A \text{ finite rep type}$$

- Question (Weyman): Does this hold for arbitrary A?

For fixed $m, n \in \mathbb{Z}$, define $A = KQ/I$ by



$$a^m = c^n = c^2 b = 0, \quad ba = cb$$

Note: each $\operatorname{mod}(A, \underline{d})$ can be interpreted as a certain space of homomorphisms of $K[x]$-modules.

Not finite type for $(m, n) \geq (4, 4)$ [Skowroński, Hoshino-Miyachi]

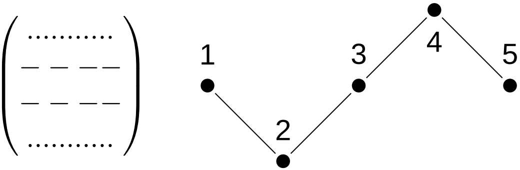
Theorem

A is DO for any (m, n).

Proof idea:

1. Matrix computations reduce to single matrix with all column ops, only some row ops between blocks:

   for example:



2. Converts to poset rep problem depending on generic Jordan types of a, c
   - [Crawley-Boevey–Schroer] simplifies which posets appear
   - Mark Kleiner's classification $\Rightarrow$ all cases are finite type $\Rightarrow$ always dense orbit.

Implication:

- Have a class of wild algebras with hope to classify generic representations.
  - "Close" to done for this family of DO algebras via Springer-type resolution, reps of commutative square, and combinatorics (work in progress).
  - Lutz Hille reports that he has a related family of DO algebras and have classified generic reps with collaborators.

Can we classify DO algebras?

Theorem

DO $\Leftrightarrow$ finite type when: A admits a preproj. component OR is special biserial OR is triangular non-distributive.

Conjecture: DO $\Leftrightarrow$ finite type for all triangular A.

Open question: Is every quotient of DO algebra also DO?

If "yes" then can prove conjecture by showing certain algebras admitting good covers are DO, using Ringel/Bongartz classification of min rep inf algebras.

- Are there generically tame algebras? Generic tame/wild dichotomy (Drozd theorem)? Generic Brauer-Thrall 2?

- There are connections/conjectures about semi-invariants running through all of this (see paper).

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Module varieties and representation type of finite-dimensional algebras
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