

# One-parameter 2-equipped posets and classification of their corepresentations

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# In memory of Alexander G. Zavadskij



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# Content

- 1 2-equipped posets
- 2 Corepresentations
- 3 2-equipped posets of one parameter type
- 4 Main Results

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- 3 2-equipped posets of one parameter type
- 4 Main Results

# Introduction

Representations of posets (partially ordered sets) are studied in algebraic representation theory.




-  P. GABRIEL & A.V. ROITER, *Representations of Finite Dimensional Algebras*, Algebra VIII, Encyclopedia of Math. Sc., **73**, Springer, (1992).
-  C.M. RINGEL, *Tame Algebras and Integral Quadratic Forms*, LMN, Springer, **1099** (1984).
-  D. SIMSON, *Linear representation of partially ordered sets and vector space categories*, Algebra, logic and applications, **4**, Gordon & Breach Sci. Publ., (1992).

Representation theory of posets starts to be studied since 70's

-  L. A. NAZAROVA & A. V. ROITER, *Representations of partially ordered sets*, Zap. Nauchn. Semin. LOMI, **28**, (1972), 5–31. In Russian, English transl. in: J.Sov.Math. 3 (1975), 585–606.
-  P. GABRIEL, *Unzerlegbare Darstellungen I*, Manuscripta Math. **6**, (1972), 71–103.



## Cases of finite and one-parameter representation type

-  M. M. KLEINER, *Partially ordered sets of finite type*, Zap. Nauchn. Semin. LOMI, **28**, (1972), 32–41. In Russian, English transl. in: J.Sov.Math. **3** (1975).
-  M. M. KLEINER, *On faithful representations of partially ordered sets of finite type*, Zap. Nauchn. Semin. LOMI, **28**, (1972), 42–59. In Russian, English transl. in: J.Sov.Math. **3** (1975).
-  V. V. OTRASHEVSKAYA, *On one-parameter partially ordered sets*, Ukr. Math. J., **28**, (1976), 334–341.




## Matrix problems over one or several fields



B. KLEMP & D. SIMSON, *Schurian  $sp$ -representation-finite right peak PI-rings and their indecomposable socle projective modules*, *J. Algebra*, **134:2**, (1990), 390-468.

# Introduction

Representations of 2-equipped posets were introduced in the late 90's.

-  A. V. ZABARILO & A. G. ZAVADSKIJ, *Representations of one-parameter equipped posets*, I, II, *Matematychni Studii*, **11** (1999), 3–16. In Russian.
-  A. G. ZAVADSKIJ, *Tame equipped posets*, *Linear Algebra Appl.*, **365** (2003), 389–465.
-  A. G. ZAVADSKIJ, *Equipped posets of finite growth*, *Representations of Algebras and Related Topics*, AMS, *Fields Inst. Comm. Ser.*, **45** (2005), 363–396.

## Classification of indecomposable corepresentations

There exists a matrix problem of mixed type over an arbitrary quadratic field extension  $F \subset G$ , in some sense dual to the representation problem.



C. RODRIGUEZ & A. G. ZAVADSKIJ, *On corepresentations of equipped posets and their differentiation*, Rev. Col. Mat. **41** (2007), 117-142.

# Introduction

## Main subjects:

- The criterion of one-parameter type for 2-equipped posets with respect to corepresentations.
- Complete list of all sincere one-parameter 2-equipped posets.
- Classification of all their indecomposable corepresentations.

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## 2-equipped posets



## 2-equipped poset



## Definition

A 2-equipped poset is a triple  $(\mathcal{P}, \leq, \triangleleft)$

- $(\mathcal{P}, \leq)$  is a poset.
- $\triangleleft$  is a binary relation over  $\mathcal{P}$  called **strong** (no necessarily order relation).
- Condition:  $x \leq y \triangleleft z$  or  $x \triangleleft y \leq z$  implies  $x \triangleleft z$ .

## 2-equipped poset



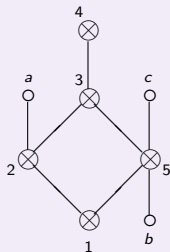
Notation:  $x \prec y$  if  $x \leq y$  but  $x \not\triangleleft y$ .

- $x \in \mathcal{P}$  is called **strong** if  $x \triangleleft x$ . ○
- $x \in \mathcal{P}$  is called **weak** if  $x \prec x$ . ⊗

# 2-equipped poset

## Example

A 2-equipped poset



Weak relations:  $1 \prec \{2, 3, 4, 5\}$ ,  $2 \prec \{3, 4\}$ ,  $5 \prec \{3, 4\}$  y  $3 \prec 4$ .

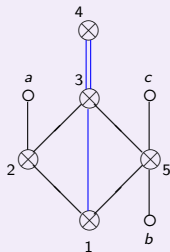
strong relations:  $a \triangleright \{1, 2\}$ ,  $c \triangleright \{1, 5, b\}$  furthermore

$4 \triangleright \{1, 2, 3, 5, b\}$  and  $1 \triangleleft 3$ .

# 2-equipped poset

## Example

A 2-equipped poset



Weak relations:  $1 \prec \{2, \cancel{3}, \cancel{4}, 5\}$ ,  $2 \prec \{3, \cancel{4}\}$ ,  $5 \prec \{3, \cancel{4}\}$  y  $3 \not\prec 4$ .  
 strong relations:  $a \triangleright \{1, 2\}$ ,  $c \triangleright \{1, 5, b\}$  furthermore  
 $4 \triangleright \{1, 2, 3, 5, b\}$  and  $1 \triangleleft 3$ .

# Corepresentations of 2-equipped posets

# $G$ -hull

## The $G$ -hull

### Definition

- $F \subset^2 G$  quadratic field extension
- $U_0$  a  $G$ -linear space and  $X \subset U_0$  a  $F$ -linear subspace.

The  **$G$ -hull** of  $X$  in  $U_0$  is  $G(X) = G X$ .

- $G X$ : is the  $G$ -space spanned by  $X$ .

- 

$$G(X) = \bigcap_{X \subset W} W, \quad W \text{ is a } G\text{-subspace.}$$

# Corepresentations

Invariant form

## Definition

A **corepresentation** of  $\mathcal{P}$  over  $(F, G)$  is any collection

$$U = (U_0, U_x : x \in \mathcal{P})$$

where  $U_0$  is a  $G$ -space containing  $F$ -spaces  $U_x$  such that

$$x \leq y \implies U_x \subset U_y,$$

$$x \triangleleft y \implies G(U_x) \subset U_y.$$

# The category of corepresentations

The category  $\text{corep } \mathcal{P}$  of corepresentations of  $\mathcal{P}$

## Definition

- **Objects**, are corepresentations of  $\mathcal{P}$ .
- **Morphism**  $\varphi$  between some objects  $U$  and  $V$  is a  $G$ -linear application  $\varphi : U_0 \rightarrow V_0$  such that  $\varphi U_x \subset V_x$  for  $x \in \mathcal{P}$ .

$U, V \in \text{corep } \mathcal{P}$  are **isomorphic** if there exist an isomorphism  $\varphi : U_0 \rightarrow V_0$  such that  $\varphi U_x = V_x$  for  $x \in \mathcal{P}$ .



# Corepresentations

## Matrix form

### Matrix corepresentation

Given a poset  $\mathcal{P}$ , a **matrix corepresentation** of  $\mathcal{P}$  over a quadratic field extension  $F \subset G$  is a rectangular matrix  $M$  over  $G$  separated into vertical stripes  $M_x$  ( $x \in \mathcal{P}$ ).

$$M = \begin{array}{c} \quad \quad \quad x \quad \quad \quad y \\ \begin{array}{|c|c|c|c|c|} \hline \dots & M_x & \dots & M_y & \dots \\ \hline \end{array} \end{array}$$

# Corepresentations

## Matrix form

The **admissible transformations** of a corepresentation  $M$  of  $\mathcal{P}$  are the following

- $G$ -elementary row transformations of the whole matrix  $M$ .
- $G(F)$ -elementary column transformations of  $M_x$  stripe if  $x$  is strong (weak).
- In case of a weak (strong) relation  $x \prec y$  ( $x \triangleleft y$ ), there are allowed column additions from the stripe  $M_x$  to the stripe  $M_y$  with coefficients in  $F(G)$ .

# Corepresentations

## Some definitions

Let  $U, V$  be corepresentations of a poset  $\mathcal{P}$ .  $U$  and  $V$  are **equivalent** if one can turn into another by means of admissible transformations.

Indecomposable and decomposable corepresentations are naturally defined.

# Corepresentations

## Some definitions

Let  $U, V$  be corepresentations of a poset  $\mathcal{P}$ .  $U$  and  $V$  are **equivalent** if one can turn into another by means of admissible transformations.

**Indecomposable** and **decomposable** corepresentations are naturally defined.

# The main problem

## Corepresentations

The **matrix problem** over the pair  $(F, G)$  consists to classify indecomposable corepresentations up to equivalence.

# One-parameter 2-equipped posets

## Definitions

### Definition

Let  $M$  be a matrix corepresentation, its **dimension** is a vector

$$d = \underline{\dim} M = (d_0; d_x \mid x \in \mathcal{P}) \quad \text{where,}$$

$d_0$ : # of rows in  $M$ ,

$d_x$ : # of columns in the stripe  $M_x$ .

# One parameter 2-equipped posets

# One-parameter 2-equipped poset

## Series

There exists an analogous definition of a matrix corepresentation of a poset  $\mathcal{P}$  over the pair of polynomial rings  $(F[t], G[t])$ .

### Series of corepresentations

A **series** of  $(F, G)$ -corepresentations is obtained from  $(F[t], G[t])$ -corepresentations by substituting any square matrix  $A$  over  $G$  for the variable  $t$  and scalar matrices  $\lambda I$  of the same size for the coefficients  $\lambda \in G$ .



# One-parameter 2-equipped posets

## Definition

A 2-equipped poset  $\mathcal{P}$  of infinite type is one-parameter (in case of infinite fields) if it has a series containing almost all indecomposable corepresentations of each dimension.

# Main Results

# Main results

One-parameter criterion of 2-equipped posets with respect to corepresentations

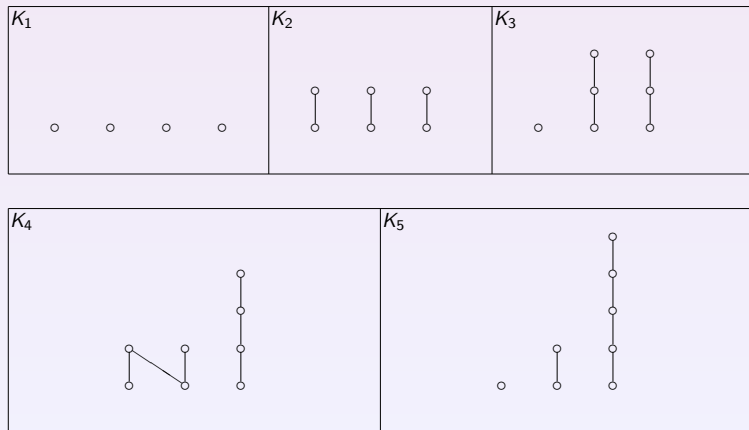
## THEOREM 1

Let  $\mathcal{P}$  be a 2-equipped poset such that  $w(\mathcal{P}) \leq 4$ . Then,  $\mathcal{P}$  is one parameter if and only if  $\mathcal{P}$  contains exactly one critical poset  $K_1, \dots, K_9$  as a subposet.

# Main results

## The critical posets

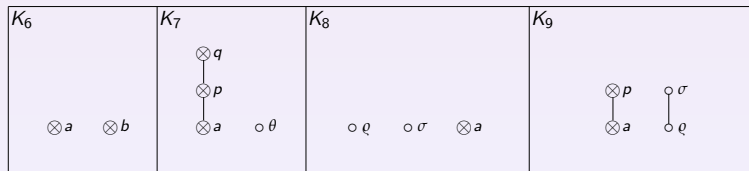
The posets  $K_1, \dots, K_5$  are the **Kleiner's** critical posets.



# One-parameter 2-equipped posets

## The critical 2-equipped posets

The posets  $K_6, \dots, K_9$  are the 2-equipped critical posets which are not trivially equipped.



# Main results

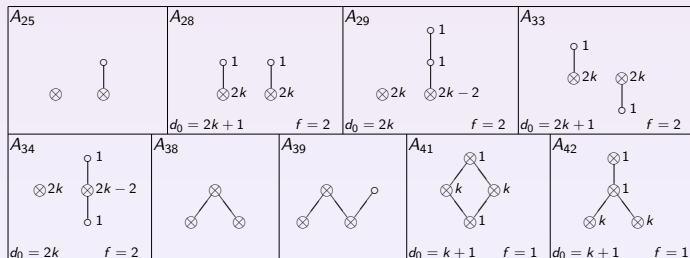
The list of sincere one-parameter 2-equipped poset

## THEOREM 2

A non trivially 2-equipped poset  $\mathcal{P}$  is one parametric if and only if  $\mathcal{P}$  is isomorphic or anti-isomorphic to one of the 28 posets  $K_6, \dots, K_9, A_{25}, \dots, A_{48}$ .

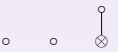
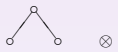
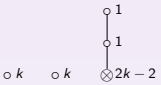
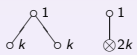
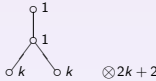
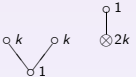
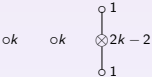
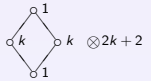

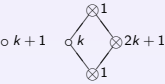
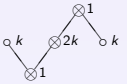
# One-parameter 2-equipped posets

The list of one parameter sincere 2-equipped posets containing  $K_6$  except critical and specials



# One-parameter 2-equipped posets

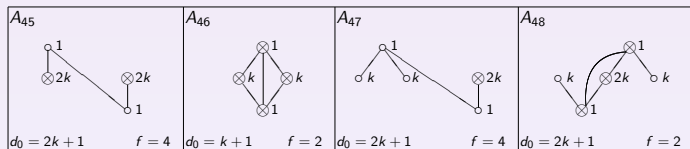
The list of one-parameter sincere 2-equipped posets containing  $K_8$  except critical and specials

$A_{26}$  $d_0 = 2k$	$A_{27}$  $d_0 = 2k$	$A_{30}$  $d_0 = 2k$ <span style="float: right;"><math>f = 2</math></span>	
$A_{31}$  $d_0 = 2k + 1$ <span style="float: right;"><math>f = 2</math></span>	$A_{32}$  $d_0 = 2k + 2$ <span style="float: right;"><math>f = 2</math></span>	$A_{35}$  $d_0 = 2k + 1$ <span style="float: right;"><math>f = 2</math></span>	$A_{36}$  $d_0 = 2k$ <span style="float: right;"><math>f = 2</math></span>
$A_{37}$  $d_0 = 2k + 2$ <span style="float: right;"><math>f = 2</math></span>	$A_{40}$  $d_0 = 2k + 2$	$A_{43}$  $d_0 = 2k + 2$ <span style="float: right;"><math>f = 1</math></span>	$A_{44}$  $d_0 = 2k + 1$ <span style="float: right;"><math>f = 1</math></span>



# One-parameter 2-equipped posets

The sincere special 2-equipped posets



## Classification of corepresentations

There were described all the sincere indecomposable corepresentations of each one-parameter 2-equipped poset, in an evident matrix form, in particular, the series of corepresentations are describe for the critical 2-equipped posets.

Series of corepresentations of critical 2-equipped posets of one parameter type

Series of corepresentations are reduced to a central homogeneous biquadratic matrix problem over an arbitrary field extension

$F \subset F(u) = G$ , solved in



A.G. ZAVADSKIJ, *A matrix problem over a central quadratic skew field extension*, Linear Algebra and its Applications, **428** (2008), 393-399.

# Main results

## Classification of indecomposable corepresentations



C. RODRIGUEZ, *On corepresentations of one parametric equipped posets*, São Paulo J. of Math. Sc. **4**, 2 (2010), 141-175.

## Main results

Classification of  $K_6$ 

tipo	dmin	paso	$f(d)$	$A$	$B$	$k_1$	$k_2$	$k_3$	$l_1$	$l_2$	$l_3$
1	$\begin{matrix} 1 \\ & 1 & 2 \end{matrix}$	$\mu$	1	$I_n$	$\overrightarrow{I}_n + \overleftarrow{\xi} I_n$	0	1	1	0	0	0
1*	$\begin{matrix} 1 \\ & 1 & 0 \end{matrix}$	$\mu$	1	$I_{n+1}$	$I_n^\uparrow + \xi I_n^\downarrow$	0	0	0	0	1	1
2	$\begin{matrix} 1 \\ & 2 & 2 \end{matrix}$	$2\mu$	2	$C_{2n+1}$	$C_{2n+1} + I_{2n}^{\rightleftharpoons}$	1	1	2	0	0	0
2*	$\begin{matrix} 1 \\ & 0 & 0 \end{matrix}$	$2\mu$	2	$I_{2n}^\downarrow$	$I_{2n}^\downarrow + C_{2n-1}^{\uparrow\uparrow}$	0	0	0	1	1	2
3 = $\tilde{3}^*$	$\begin{matrix} 1 \\ & 2 & 0 \end{matrix}$	$2\mu$	2	$\overrightarrow{I}_{2n+1} + \overleftarrow{\xi} I_{2n+1}$	$I_{2n}^\uparrow$	1	0	0	0	1	0
4 = $4^*$	$\begin{matrix} 2 \\ & 2 & 2 \end{matrix}$	$2\mu$	0	$P_{2n}$	$R_{2n}^\uparrow$	0	1	0	0	1	0
5 = $5^*$	$\begin{matrix} 1 \\ & 1 & 1 \end{matrix}$	$\mu$	0	$I_n$	$I_n + \xi J_n^+(0)$	0	0	1	0	0	1
6 = $6^*$	$\begin{matrix} 1 \\ & 1 & 1 \end{matrix}$	$\mu$	0	$I_n$	$\tilde{\xi} I_n + X$	0	0	0	0	0	0

## Remark

The  $X$  block represent an indecomposable Frobenius block over  $F$ .

## Main results

Example  $(K_6 - 1^*)$ 

1	$\zeta$
1	1

 ${}_2(K_6 - 1^*)$  $d = (2, 2, 1)$ 

1	$\zeta$
1	1 $\zeta$
1	1

 ${}_3(K_6 - 1^*)$  $d = (3, 3, 2)$ 

1	$\zeta$
1	1 $\zeta$
1	1 $\zeta$
1	1

 ${}_4(K_6 - 1^*)$  $d = (4, 4, 3)$

# Main results

Example of the series of  $K_6$  for the case  $(\mathbb{R}, \mathbb{C})$ :  $\begin{bmatrix} 1_n & i1_n + X \end{bmatrix}$ , where  $X$  is an indecomposable Fröbenius block.

For  $d = (1, 1, 1)$

$$\begin{bmatrix} 1 & i + 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & i + 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & i + 4 \end{bmatrix}, \dots$$

For  $d = (2, 2, 2)$

$$\begin{bmatrix} 1 & i & -3 \\ & 1 & i \end{bmatrix}, \quad \begin{bmatrix} 1 & i & -1 \\ & 1 & i-1 \end{bmatrix}, \quad \begin{bmatrix} 1 & i & -2 \\ & 1 & i-1 \end{bmatrix}, \dots$$

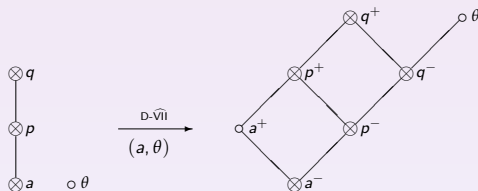
## Main results

Classification of  $K_7$ 

$(K_7 - 1) = \text{Int}(K_6 - 1)$	$(K_7 - 1^*) = \text{Int}(K_6 - 1^*)$
$(K_7 - 2) = \text{Int}(K_6 - \bar{1})$	$(K_7 - 2^*) = \text{Int}(K_6 - \bar{1}^*)$
$(K_7 - 3) = \text{Int}(A_{38} - 1)$	$(K_7 - 3^*) = \text{Int}(A_{38^*} - 1)$
$(K_7 - 4) = \text{Int}(A_{38} - 2)$	$(K_7 - 4^*) = \text{Int}(A_{38^*} - 2)$
$(K_7 - 5) = \text{Int}(A_{38} - \bar{2})$	$(K_7 - 5^*) = \text{Int}(A_{38^*} - \bar{2})$
$(K_7 - 6) = \text{Int}(A_{38} - 3)$	$(K_7 - 6^*) = \text{Int}(A_{38^*} - 3)$
$(K_7 - 7) = \text{Int}(A_{25} - 1)$	$(K_7 - 7^*) = \text{Int}(A_{25^*} - 1)$
$(K_7 - 8) = \text{Int}(A_{39} - 1)$	$(K_7 - 8^*) = \text{Int}(A_{39^*} - 1)$
$(K_7 - 9) = \text{Int}(A_{39} - 2)$	$(K_7 - 9^*) = \text{Int}(A_{39^*} - 2)$
$(K_7 - 10) = \text{Int}(A_{39} - 3)$	$(K_7 - 10^*) = \text{Int}(A_{39^*} - 3)$
$(K_7 - 11) = \text{Int}(A_{39} - 4)$	$(K_7 - 11^*) = \text{Int}(A_{39^*} - 4)$
$(K_7 - 12) = \text{Int}(A_{39} - 5)$	$(K_7 - 12^*) = \text{Int}(A_{39^*} - 5)$

$(K_7 - 13) = \text{Int}(K_6 - 2)$	$(K_7 - 13^*) = \text{Int}(K_6 - 2^*)$
$(K_7 - 14) = \text{Int}(K_6 - 3)$	$(K_7 - 14^*) = \text{Int}(K_6 - 3^*)$
$(K_7 - 15) = \text{Int}(A_{38} - 4)$	$(K_7 - 15^*) = \text{Int}(A_{38^*} - 4)$
$(K_7 - 16) = \text{Int}(A_{25} - 2)$	$(K_7 - 16^*) = \text{Int}(A_{25^*} - 2)$
$(K_7 - 17) = \text{Int}(A_{25} - 3)$	$(K_7 - 17^*) = \text{Int}(A_{25^*} - 3)$
$(K_7 - 18) = \text{Int}(A_{25} - 4)$	$(K_7 - 18^*) = \text{Int}(A_{25^*} - 4)$
$(K_7 - 19) = \text{Int}(A_{39} - 6)$	$(K_7 - 19^*) = \text{Int}(A_{39^*} - 6)$
$(K_7 - 20) = \text{Int}(A_{39} - 7)$	$(K_7 - 20^*) = \text{Int}(A_{39^*} - 7)$
$(K_7 - 21) = \text{Int}(A_{39} - 8)$	$(K_7 - 21^*) = \text{Int}(A_{39^*} - 8)$
$(K_7 - 22) = \text{Int}(A_{39} - 9)$	$(K_7 - 22^*) = \text{Int}(A_{39^*} - 9)$
$(K_7 - 23) = \text{Int}(A_{39} - 10)$	$(K_7 - 23^*) = \text{Int}(A_{39^*} - 10)$
$(K_7 - 24) = \text{Int}(K_6 - 4)$	
$(K_7 - 25) = \text{Int}(A_{33})$	



The serie of  $K_7$ 

The serie

$a$	$p$	$q$	$\theta$
$I_n$	$X$	$I_n$	$I_n$
$I_n$	$\tilde{\zeta} I_n$		

where  $X$  is an indecomposable Frobenius block over  $F$ .

## Main results

Classification of indecomposable corepresentations of  $K_8$ 

$$M_U \text{ has the matrix } \begin{array}{c|cc} & \begin{array}{c} \rho \\ \sigma \\ \alpha \end{array} & \\ \hline \begin{array}{c} R \\ 0 \end{array} & \begin{array}{cc} 0 & A_1 \ A_2 \\ S & A'_1 \ A'_2 \end{array} \end{array}$$

tipo	dmin	paso	$f(d)$	$R$	$A_1$	$A_2$	$S$	$A'_1$	$A'_2$
1	<sup>2</sup> <sub>1 1 3</sub>	$\mu$	1	$I_n$	$\overrightarrow{I_n}$	$\xi I_n$	$I_n$	$\overleftarrow{I_n}$	$\xi I_n$
1*	<sup>2</sup> <sub>1 1 1</sub>	$\mu$	1	$I_{n+1}$	$I_n^\downarrow$	$\xi I_{n+1}$	$I_{n+1}$	$I_n^\uparrow$	$\xi I_{n+1}$
2 = 2*	<sup>1</sup> <sub>1 0 1</sub>	$\mu$	1	$I_{n+1}$	$I_{n+1}$	$\xi I_n^\uparrow$	$I_n$	$\overrightarrow{I_n}$	$\xi I_n$
3	<sup>1</sup> <sub>1 0 2</sub>	$\mu$	2	$I_{n+1}$	$I_{n+1}$	$\xi I_{n+1}$	$I_n$	$\overrightarrow{I_n}$	$\xi \overleftarrow{I_n}$
3*	<sup>1</sup> <sub>0 1 0</sub>	$\mu$	2	$I_n$	$I_n$	$\xi I_n$	$I_{n+1}$	$I_n^\uparrow$	$\xi I_n^\downarrow$
4 = 4*	<sup>2</sup> <sub>1 1 2</sub>	$\mu$	0	$I_n$	$I_n$	$\xi J_n(0)$	$I_n$	$I_n$	$\xi I_n$
5 = 5*	<sup>2</sup> <sub>1 1 2</sub>	$\mu$	0	$I_n$	$I_n$	$\xi J_n(1)$	$I_n$	$I_n$	$\xi I_n$
6 <sub>i</sub>	<sup>2</sup> <sub>1 1 2</sub>	$\mu$	0	$I_n$	$I_n$	$\xi I_n$	$I_n$	$[I_n - \overline{C(f)}]$	$\xi[I_n + \overline{C(f)}]$
6 <sub>ii</sub>	<sup>2</sup> <sub>1 1 2</sub>	$\mu$	0	$I_n$	$I_n$	$\xi I_n$	$I_n$	$[I_n + \overline{C(f)}]$	$[\xi I_n + \xi \overline{C(f)}]$
6 <sub>iii</sub>	<sup>2</sup> <sub>1 1 2</sub>	$\mu$	0	$I_n$	$I_n$	$\xi I_n$	$I_n$	$[\overline{C(f)}]$	$\xi[I_n + C(f)]$

## Main results

Classification of indecomposable corepresentations of  $K_8$ 

$\varrho$	$\sigma$	$\mathfrak{a}$	
$1\ 0\ \dots\ 0$	$1\ 0\ \dots\ 0$	$T_1$	$T_2$
$R$	$0$	$A_1$	$A_2$
$0$	$S$	$A'_1$	$A'_2$

 $M_U$  has the matrix form

tipo	dmin	paso	$f(d)$	$R$	$A_1$	$A_2$	$S$	$A'_1$	$A'_2$	$T_1$	$T_2$	
7	$^1$	$1\ 1\ 1$	$\mu$	1	$\overleftarrow{l}_n$	$\overrightarrow{l}_n$	$\xi \overleftarrow{l}_n$	$\overleftarrow{l}_n$	$\overleftarrow{l}_n$	$\xi \overrightarrow{l}_n$	$0\ 0\ \dots\ 0$	$1\ 0\ \dots\ 0$
8	$^1$	$1\ 1\ 2$	$\mu$	2	$\overleftarrow{l}_n$	$\overrightarrow{l}_n$	$\xi \overleftarrow{l}_n$	$\overleftarrow{l}_n$	$\overleftarrow{l}_n$	$\xi \overrightarrow{l}_n$	$0\ \dots\ 0\ 1$	$0\ \dots\ 0\ \xi$
9	$^2$	$1\ 2\ 2$	$\mu$	2	$\overleftarrow{l}_n$	$\overrightarrow{l}_n$	$\xi \overleftarrow{l}_n$	$\overleftarrow{l}_{n+1}$	$l_{n+1}$	$\xi \overrightarrow{l}_{n+1}$	$0\ 0\ \dots\ 0$	$1\ 0\ \dots\ 0$
10	$^1$	$1\ 1\ 0$	$\mu$	2	$\overleftarrow{l}_n$	$l_n$	$\xi J_n(1)$	$\overleftarrow{l}_n$	$l_n$	$\xi \overrightarrow{l}_n$	$0\ 0\ \dots\ 0$	$1\ 0\ \dots\ 0$

$\varrho$	$\sigma$	$\mathfrak{a}$	
$R$	$0$	$A_1$	$A_2$
$0$	$S$	$A'_1$	$A'_2$
$0\ 0\ \dots\ 0$	$0\ 0\ \dots\ 0$	$L_1$	$L_2$

 $M_U$  has the matrix form

tipo	dmin	paso	$f(d)$	$R$	$A_1$	$A_2$	$S$	$A'_1$	$A'_2$	$L_1$	$L_2$	
7*	$^1$	$0\ 0\ 1$	$\mu$	1	$l_n$	$l_n$	$\xi \overleftarrow{l}_n$	$l_n$	$l_n$	$\xi \overrightarrow{l}_n$	$1\ \dots\ 1\ 1$	$\xi \ \dots\ \xi\ \xi$
8*	$^1$	$0\ 0\ 0$	$\mu$	2	$l_n$	$J_n(0)$	$\xi \overleftarrow{l}_n$	$l_n$	$l_n$	$\xi J_n(0)$	$1\ \dots\ 1\ 1$	$\xi \ \dots\ \xi\ \xi$
9*	$^2$	$1\ 0\ 2$	$\mu$	2	$l_{n+1}$	$l_{n+1}$	$\xi J_{n+1}(0)$	$l_n$	$\overleftarrow{l}_n$	$\xi \overleftarrow{l}_n$	$1\ \dots\ 1\ 1$	$\xi \ \dots\ \xi\ \xi$
10*	$^1$	$0\ 0\ 2$	$\mu$	2	$l_n$	$\overrightarrow{l}_n$	$\xi \overrightarrow{l}_n + \xi \overleftarrow{l}_n$	$l_n$	$\overrightarrow{l}_n$	$\xi \overrightarrow{l}_n$	$0\ \dots\ 0\ 1$	$0\ \dots\ 0\ \xi$

$$\begin{array}{l}
 \text{tipo} \\
 K_8 - 11 = 11^*
 \end{array}
 M_U \leftrightarrow
 \begin{array}{c}
 \varrho \quad \sigma \quad \mathfrak{a} \\
 \begin{array}{|c|c|c|}
 \hline
 l_n & l_n & 0 \ \xi \overrightarrow{l}_n \\
 \hline
 J_n^+(0) & 0 & l_n \ \xi \overrightarrow{l}_n \\
 \hline
 \end{array}
 \end{array}
 \begin{array}{l}
 \text{dmin} : \quad ^2 \quad 1 \ 1 \ 2 \\
 \text{paso} \quad \mu \\
 f(d) = 0
 \end{array}$$

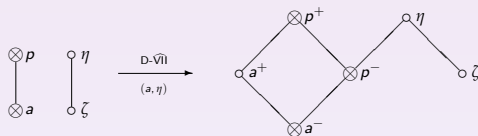
# Main results

## Classification of indecomposable corepresentations of $K_9$

$(K_9 - 1) = \text{Int}(K_8 - 1)$	$(K_9 - 1^*) = \text{Int}(A_{40} - 6)$
$(K_9 - 2) = \text{Int}(K_8 - 1^*)$	$(K_9 - 2^*) = \text{Int}(A_{40^*} - 6)$
$(K_9 - 3) = \text{Int}(K_8 - 2)$	$(K_9 - 3^*) = \text{Int}(A_{40^*} - 3)$
$(K_9 - 4) = \text{Int}(K_8 - 2^*)$	$(K_9 - 4^*) = \text{Int}(A_{40} - 3)$
$(K_9 - 5) = \text{Int}(K_8 - 7)$	$(K_9 - 5^*) = \text{Int}(A_{40} - 1)$
$(K_9 - 6) = \text{Int}(K_8 - 7^*)$	$(K_9 - 6^*) = \text{Int}(A_{40^*} - 1)$
$(K_9 - 7) = \text{Int}(A_{40} - 2)$	$(K_9 - 7^*) = \text{Int}(A_{40^*} - 7)$
$(K_9 - 8) = \text{Int}(A_{40} - 7)$	$(K_9 - 8^*) = \text{Int}(A_{40^*} - 2)$
$(K_9 - 9) = \text{Int}(A_{40} - 11)$	$(K_9 - 9^*) = \text{Int}(A_{40^*} - 13)$
$(K_9 - 10) = \text{Int}(A_{40} - 12)$	$(K_9 - 10^*) = \text{Int}(A_{40^*} - 11)$
$(K_9 - 11) = \text{Int}(A_{40} - 13)$	$(K_9 - 11^*) = \text{Int}(A_{40^*} - 12)$
$(K_9 - 12) = \text{Int}(A_{40} - 5)$	
$(K_9 - 13) = \text{Int}(A_{40^*} - 5)$	

$(K_9 - 14) = \text{Int}(K_8 - 3)$	$(K_9 - 14^*) = \text{Int}(K_8 - \bar{9}^*)$
$(K_9 - 15) = \text{Int}(K_8 - 3^*)$	$(K_9 - 15^*) = \text{Int}(K_8 - \bar{9})$
$(K_9 - 16) = \text{Int}(K_8 - \bar{3})$	$(K_9 - 16^*) = \text{Int}(A_{40} - 10)$
$(K_9 - 17) = \text{Int}(K_8 - \bar{3}^*)$	$(K_9 - 17^*) = \text{Int}(A_{40^*} - 10)$
$(K_9 - 18) = \text{Int}(K_8 - 8)$	$(K_9 - 18^*) = \text{Int}(A_{40} - 8)$
$(K_9 - 19) = \text{Int}(K_8 - 8^*)$	$(K_9 - 19^*) = \text{Int}(A_{40^*} - 8)$
$(K_9 - 20) = \text{Int}(K_8 - 9)$	$(K_9 - 20^*) = \text{Int}(A_{40} - 4)$
$(K_9 - 21) = \text{Int}(K_8 - 9^*)$	$(K_9 - 21^*) = \text{Int}(A_{40^*} - 4)$
$(K_9 - 22) = \text{Int}(A_{40} - 9)$	$(K_9 - 22^*) = \text{Int}(A_{40^*} - 9)$
$(K_9 - 23) = \text{Int}(A_{40} - 14)$	$(K_9 - 23^*) = \text{Int}(A_{40^*} - 15)$
$(K_9 - 24) = \text{Int}(A_{40} - 15)$	$(K_9 - 24^*) = \text{Int}(A_{40^*} - 14)$
$(K_9 - 25) = \text{Int}(K_8 - 10)$	
$(K_9 - 26) = \text{Int}(K_8 - 10^*)$	

# The serie of $K_9$



## The serie

$\overbrace{\quad}^a$		$\overbrace{\quad}^p$		$\zeta$	$\eta$
		A	B	$I_n$	
	$I_n$	$I_n$	$\zeta I_n$		$I_n$
$I_n$	$\zeta I_n$				

$A = [I_n - \overline{C(f)}]$  and  $B = \zeta[I_n + \overline{C(f)}]$ , if  $F \subset G$  is separable and  $\text{car} \neq 2$ ;

$A = [I_n + \overline{C(f)}]$  and  $B = [\zeta I_n + \zeta \overline{C(f)}]$ , if  $F \subset G$  separable and  $\text{car} = 2$ ;

$A = [\overline{C(f)}]$  and  $B = \zeta[I_n + C(f)]$ , if  $F \subset G$  is inseparable.

# Thanks