Module categories for elementary abelian p-groups and generalized Beilinson algebras

Julia Worch

University of Kiel

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We confine our investigations to elementary abelian p-groups.

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- $I := (X_1, \ldots, X_r) \subseteq k[X_1, \ldots, X_r]$ is the ideal generated by polynomials of degree 1.

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- $\operatorname{CR}(kE_r)$, $\operatorname{EIP}(kE_r)$ and $\operatorname{EKP}(kE_r)$ are the corresponding full subcategories of $\operatorname{mod}(kE_r)$.

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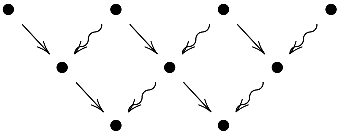

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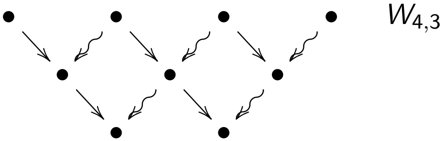

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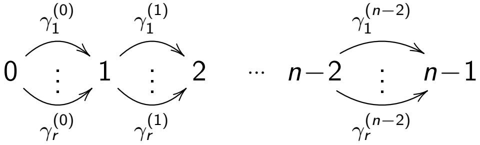

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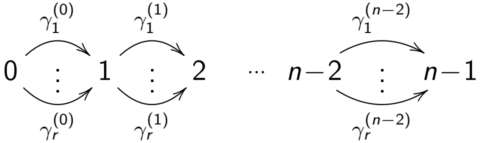

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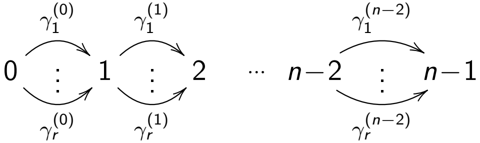

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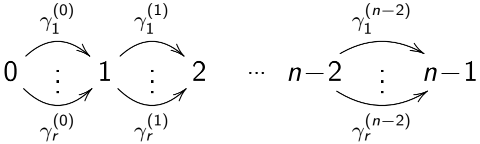

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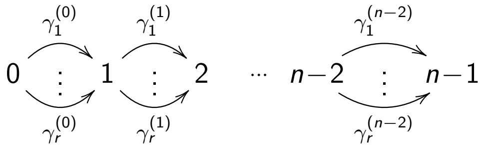

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In particular, there are no non-trivial maps EIP$(n, r) \to$ EKP(n, r).

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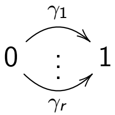

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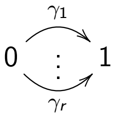

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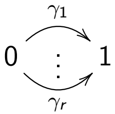

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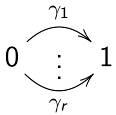

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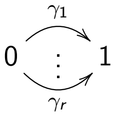

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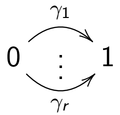

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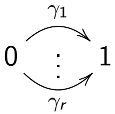

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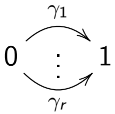

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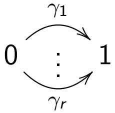

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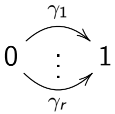

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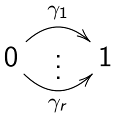

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  - $r > 2$: $\mathbb{Z}A_\infty$-components

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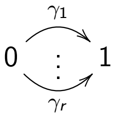
- $B(2,r)$ is the path algebra of the r-Kronecker



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Regular equal images modules

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Let $\mathcal{C}$ be a regular component of Γ_r, $r > 2$. Then $\mathrm{EIP}(2, r) \cap \mathcal{C}$ and $\mathrm{EKP}(2, r) \cap \mathcal{C}$ are non-empty disjoint cones.

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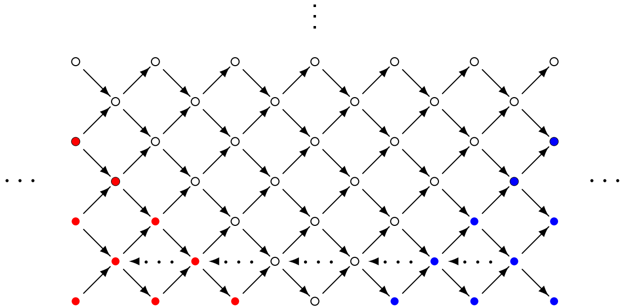
Let $\mathcal{C}$ be a regular component of Γ_r, $r > 2$. Then $\mathrm{EIP}(2, r) \cap \mathcal{C}$ and $\mathrm{EKP}(2, r) \cap \mathcal{C}$ are non-empty disjoint cones. The size of the gap $\mathcal{W}(\mathcal{C}) \in \mathbb{N}_0$ between these cones is an invariant of $\mathcal{C}$.



Modules of constant rank

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Examples:

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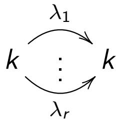

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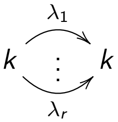

satisfies $\mathcal{W}(\mathcal{C}_\lambda) = 1$.