SFB–Seminar

Am Montag, den 9.12.1996 findet folgender Vortrag statt:

The Localised Euler class
or
how to count zeros with multiplicity

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Der Vortrag beginnt um 16:00 c.t. Uhr im Raum V2-205.

Abstract

A vector field v on an even dimensional sphere has 2 zeros, at least when counted with the proper multiplicity. For general vector fields, i.e., those with isolated non-degenerate zeros, the algebraic number of zeros $\mathbb{Z}(v)$ is just the number of zeros counted with orientation. The algebraic number of zeros is stable under small perturbations of the vector field. Thus one could define $\mathbb{Z}(v)$ for all vector fields by choosing a small perturbation Δv such that $v + \Delta v$ is general, setting $\mathbb{Z}(v) = \mathbb{Z}(v + \Delta v)$, and proving this is independent of choices. But in fact $\mathbb{Z}(v)$ is naturally *the Euler class* of the tangent bundle which has a direct construction in algebraic topology. This realisation is useful in computations: it gives the number 2, the Euler characteristic of the sphere.

On a non-compact manifold like $\mathbb{R}^n$ one can still define an algebraic number of zeros with good deformation properties, but it depends on the behaviour of the vector field "at infinity". For example, the number of zeros of a holomorphic function on a disk extending continuously and non-vanishing to the bounding circle, is the winding number of the restriction to this circle. This is still true if we replace "holomorphic function" by "continuous complex valued function" and "number of zeros" by "algebraic number of zeros".

Donaldson showed how to count the algebraic number of solutions of certain non-linear elliptic PDE's by a procedure similar to the perturbation definition of $\mathbb{Z}(v)$ but applied to an infinite dimensional manifold.

We explain that all these examples can be understood in terms of the *localised Euler class.*

Es gibt Kaffee und Tee im Blauen Salon schon um 15:30 Uhr.

G. Röhrle