RICH COLORINGS WITH LOCAL CONSTRAINTS

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Abstract

A new hypergraph coloring problem is introduced by defining $N(H,e)$ as the maximal number of colors in a vertex coloring of a hypergraph $H=(V,\mathcal{E})$, which has not more than e different colors in every edge. Our main results concern the asymptotic behaviour of this quantity for the uniform hypergraph $H(n,\ell,k)=(\mathcal{V}(n,\ell),\mathcal{E}(n,\ell,k))$ with vertex set $\mathcal{V}(n,\ell)=\binom{\Omega_n}{\ell}$ for $\Omega_n=\{1,2,\dots,n\}$ and edge set $\mathcal{E}(n,\ell,k)=\left\{E=\binom{A}{\ell}:A\in\binom{\Omega_n}{k}\right\}$. In case $\ell=2$ there are connections to Turan's graph.