ON SETS OF WORDS WITH PAIRWISE COMMON LETTER IN DIFFERENT POSITIONS

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1. Introduction and results[1]

For a finite (or infinite) alphabet $X_\alpha = \{1, 2, \ldots, \alpha\}$ we consider the set of words of length n

$$X_\alpha^n = \{x^n = (x_1, x_2, \ldots, x_n) : x_t \in X_\alpha \text{ for } t = 1, 2, \ldots, n\} \tag{1.1}$$

and also its subset W_α^n of words without repitition of letters, that is,

$$W_\alpha^n = \{x^n = (x_1, x_2, \ldots, x_n) | x_t \in X_\alpha \text{ and } x_s \neq x_t \text{ for } s \neq t\} \ . \tag{1.2}$$

We describe now an extremal problem, which was mentioned by Mullin at the meeting Designs and Codes, Oberwolfach April 1990, and was said to be of interest to computer scientists.

The words x^n and y^n are in "good relation", if for some $s \neq t$ $x_s = y_t$. For this relation we write $x^n \swarrow\!\searrow y^n$.

The set $F \subset X_\alpha^n$ is good, if for all $x^n, y^n \in F$ $x^n \swarrow\!\searrow y^n$.

Denoting the family of all good sets in W_α^n by $\mathcal{F}_\alpha^n$ the quantity of interest is

$$f_\alpha^n = \max\{|F| : F \in \mathcal{F}_\alpha^n\} \ . \tag{1.3}$$

Generally speaking the determination of this number constitutes an extremal problem in a (growing) class of similar problems whose prototype or historically first candidate is the intersection problem of [1].

Clearly, it is certainly also meaningful to study $\mathcal{G}_\alpha^n$, the family of all good sets in X_α^n , and the quantity

$$g_\alpha^n = \max\{|E| : E \in \mathcal{G}_\alpha^n\} \ . \tag{1.4}$$

The functions f_α^n and g_α^n are rather complex. We present here results for the first two non–trivial configurations of the parameters α and n , namely the cases $n = \alpha - 1$ and $n = 3$. Also, we have a limit theorem for α tending to infinity.

Specifically, we have the following results.

Theorem 1.

$f_\alpha^{\alpha-1} = \frac{1}{2}|W_\alpha^{\alpha-1}| = \frac{1}{2}\alpha!$.

Moreover, we determine all optimal configurations.

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[1] After these results were obtained we received a preprint "Sets of properly separated permutations" by R.C. Mullin, D.R. Stinson, and W.D. Wallis. The relation to our work is this:

Our Theorem 2 and their Theorem 2.2 are identical. Our Theorem 1 goes beyond their Theorem 1.1, because we determine all optimal configurations. Otherwise the papers have no overlap. In particular, our Theorems 3 and 4 are new.