THE ASYMPTOTIC BEHAVIOUR OF DIAMETERS IN THE AVERAGE

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Abstract

In 1975 Ahlswede and Katona posed the following average distance problem ([1], page 10): For every cardinality $a \in \{1, \dots, 2^n\}$ determine subsets A of $\{0,1\}^n$ with $\#A = a$, which have **minimal** average **inner** Hamming distance. Recently Althöfer and Sillke gave an exact solution of this problem for the central value $a = 2^{n-1}$.

Here we present nearly optimal solutions for $a = 2^{\lambda n}$ with $0 < \lambda < 1$: Asymptotically it is not possible to do better than choosing

$$A_n = \big\{(x_1, \dots, x_n) \mid \sum_{t=1}^{n} x_t = \lfloor \alpha n \rfloor \big\},$$

where $\lambda = -\alpha \log \alpha - (1-\alpha)\log(1-\alpha)$.

Next we investigate the following more general problem, which occurs for instance in the construction of good "Write–Efficient–Memories [WEMs]":

Given any finite set M with an arbitrary cost function $d : M \times M \to \mathbb{R}$, the corresponding sum type cost function $d_n : M^n \times M^n \to \mathbb{R}$ is defined by $d_n\big((x_1, \dots, x_n), (y_1, \dots, y_n)\big) = \sum_{i=1}^{n} d(x_t, y_t)$. The task is to find sets A_n of a given cardinality, which minimize the average inner cost $\frac{1}{(\#A_n)^2} \sum_{a \in A_n} \sum_{a' \in A_n} d_n(a, a')$. We prove that asymptotically optimal sets can be constructed by using "mixed typical sequences" with at most two different local configurations. As a non–trivial example we look at the Hamming distance for $M = \{1, \dots, m\}$ with $m \geq 3$.