On multi–user write–efficient memories

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Abstract

Continuing our earlier work on write–efficient memories (WEM) we introduce now models, where several persons use the same storage device. At any time instant exactly one of a prescribed set of users has access to the memory, but there is no protocol, which determines the moving order. Among the constraints we analyse the most interesting one is a complete privacy protection. While a user stores new data he has to guarantee that those of the others don't get distorted.

This leads to fascinating new coding problems. We provide several code constructions and also abstract performance bounds.

Index Terms: Multi–user memories, WEM, WUM, privacy constraints, code constructions, identification.

1 Introduction: Multi–user memories with constraints on privacy, hierarchy and technology

Write–efficient memories (WEM) were introduced in [2] as a model for storing and updating information on a rewritable medium. There is a cost $\varphi : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_\infty$ assigned to changes of letters. A collection of subsets $\mathcal{C} = \{C_i : 1 \le i \le M\}$ of $\mathcal{X}^n$, called clouds, is an (n, M, D) WEM code, if $C_i \cap C_j = \varnothing$ for all $i \neq j$ and if

$$D_{\max} = \max_{1 \le i,j \le M} \max_{x^n \in C_i} \min_{y^n \in C_j} \sum_{t=1}^{n} \varphi(x_t, y_t) \le D. \tag{1.1}$$

$D_{\max}$ is called the maximal correction cost with respect to the given cost function. The performance of a code $\mathcal{C}$ can also be measured by two parameters, namely, the maximal cost per letter $d_{\mathcal{C}} = n^{-1}\, D_{\max}$ and the rate of the size $r_{\mathcal{C}} = n^{-1} \log M$. The rate achievable with a maximal per letter cost d is thus

$$R(d) = \sup_{\mathcal{C}: d_{\mathcal{C}} \le d} r_{\mathcal{C}}. \tag{1.2}$$

This is the most basic quantity (the storage capacity) of a WEM $(\mathcal{X}^n, \varphi^n)_{n=1}^{\infty}$.

For a WEM code $\mathcal{C}$ the average correction cost D_{ave} can be defined as

$$D_{\text{ave}} = \frac{1}{M^2} \sum_{1 \le i,j \le M} \frac{1}{|C_i|} \sum_{x^n \in C_i} \min_{y^n \in C_j} \sum_{t=1}^{n} \varphi(x_t, y_t) \tag{1.3}$$

and the average cost per letter can be defined as

$$\overline{d}_{\mathcal{C}} = n^{-1} D_{\text{ave}}. \tag{1.4}$$

The rate achievable with an average per letter cost d is thus

$$\overline{R}(d) = \sup_{\mathcal{C}: \overline{d}_{\mathcal{C}} \le d} r_{\mathcal{C}}. \tag{1.5}$$

The main result of [2] is

$$\overline{R}(d) = R(d) = \sup_{P_{XY} \in \mathcal{P}_d} H(Y|X), \tag{1.6}$$

where $\mathcal{P}_d$ is a set of bivariate distributions

$$\mathcal{P}_d = \big\{ P_{XY} \in \mathcal{P}(\mathcal{X} \times \mathcal{X}) : P_X = P_Y, \mathbb{E}\varphi(X, Y) \le d \big\}, \tag{1.7}$$

P_{XY} is the distribution of a pair of random variables (X, Y) with values in $\mathcal{X} \times \mathcal{X}$, P_X (resp. P_Y) is the distribution of X (resp. Y), $H(Y|X)$ is the conditional entropy of Y given X, and $\mathbb{E}$ denotes the expected value.