CROSS–DISJOINT PAIRS OF CLOUDS IN THE INTERVAL LATTICE

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Abstract

Let $\mathcal{I}_n$ be the lattice of intervals in the Boolean lattice $\mathcal{L}_n$. For $\mathcal{A}, \mathcal{B} \subset \mathcal{I}_n$ the pair of clouds $(\mathcal{A}, \mathcal{B})$ is cross–disjoint, if $I \cap J = \phi$ for $I \in \mathcal{A}$, $J \in \mathcal{B}$. We prove that for such pairs $|\mathcal{A}||\mathcal{B}| \leq 3^{2n-2}$ and that this bound is best possible.

Optimal pairs are up to obvious isomorphisms unique. The proof is based on a new bound on cross intersecting families in $\mathcal{L}_n$ with a weight distribution. It implies also an Intersection Theorem for multisets of Erdös and Schönheim [9].