Minimizing the condition number of a positive definite matrix by completion

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Abstract

We consider the problem of minimizing the spectral condition number of a positive definite matrix by completion:

$$\min\{cond(\begin{bmatrix} A & B^H \\ B & X \end{bmatrix}) : \begin{bmatrix} A & B^H \\ B & X \end{bmatrix} \text{positive definite}\},$$

where A is an $n \times n$ Hermitian positive definite matrix, B a $p \times n$ matrix and X is a free $p \times p$ Hermitian matrix. We reduce this problem to an optimization problem for a convex function in one variable. Using the minimal solution of this problem we characterize the complete set of matrices that give the minimum condition number.

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