ERASURE, LIST AND DETECTION ZERO–ERROR CAPACITIES FOR LOW NOISE AND A RELATION TO IDENTIFICATION

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Abstract

For the discrete memoryless channel $(\mathcal{X},\mathcal{Y},W)$ we give characterisations of the zero–error erasure capacity C_{er} and the zero–error average list size capacity $C_{\text{a}\ell}$ in terms of limits of suitable information resp. divergence quantities (Theorem 1). However, they don't "single–letterize".

Next we assume that $\mathcal{X}\subset\mathcal{Y}$ and $W(x|x)>0$ for all $x\in\mathcal{X}$, and we associate with W the low noise channel W_ε, where for $\mathcal{Y}^+(x)=\{y:W(y|x)>0\}$

$$W_\varepsilon(y|x)=\begin{cases}1, & \text{if } y=x \text{ and } |\mathcal{Y}^+(x)|=1\\ 1-\varepsilon, & \text{if } y=x \text{ and } |\mathcal{Y}^+(x)|>1\\ \frac{\varepsilon}{|\mathcal{Y}^+(x)|-1}, & \text{if } y\neq x.\end{cases}$$

Our Theorem 2 says that as ε tends to zero the capacities $C_{\text{er}}(W_\varepsilon)$ and $C_{\text{a}\ell}(W_\varepsilon)$ relate to the zero–error detection capacity $C_{\text{de}}(W)$.

Our third result is a seemingly basic contribution to the Theory of Identification via Channels. We introduce the (second order) identification capacity C_{oid} for identification codes with zero misrejection probability and misacceptance probability tending to zero. Our Theorem 3 says that C_{oid} equals the zero–error erasure capacity for transmission C_{er}.

Key words: Zero–error erasure capacity, zero–error average list size capacity, zero–error detection capacity, identification with zero misrection probability, low noise channels.

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