MESSY BROADCASTING IN NETWORKS

BY

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Dedicated to James L. Massey on the occasion of his sixtieth birthday.

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An apology to all friends of J.L. M.

Among the many discoveries Jim has made until now one observation (Taschkent 1984) is that R.A. seldom does what most people expect him to do. Here we have made an attempt to give a contribution which has no relation to anything J.L. M. ever did.

However, it relates to him. It is in the spirit of the following question: "Which travel time to Zürich can an absentminded mathematician guarantee, if at any station he chooses any train in any available direction without going to the same city twice?"

1. Introduction

Broadcasting refers to the process of message dissemination in a communication network whereby a message, originated by one of the members, is transmitted to all members of the network.

A communication network is a connected graph $G = (V, E)$, where V is a set of vertices (members) and E is a set of edges. Transmission of the message from the originator to all members is said to be broadcasting, if the following conditions hold:

1) Any transmission of information requires a unit of time.
2) During one unit of time every informed vertex (member) can transmit information to one of its neighboring vertices (members).

The classical model.

For a $u \in V$ we define the broadcast time $t(u)$ of vertex u as the minimum number of time units required to complete broadcasting starting from vertex u. We denote by $t(G) = \max_{u \in V} t(u)$ the broadcast time of graph G. It is easy to see that for any connected graph G $t(G) \geq \lceil \log_2 n \rceil$, where $n = |V|$, since during each time unit the number of informed vertices can at most be doubled.

A minimal broadcast graph (MBG) is a graph with n vertices, in which a message can be broadcasted in $\lceil \log_2 n \rceil$ time units.

The broadcast function β assigns to n as value $\beta(n)$ the minimum number of edges in a MBG on n vertices. Presently exact values of $\beta(n)$ are known only for two infinite sets of parameters of MBG's, namely, for $\{n = 2^m : m = 1, 2, 3, \dots\}$ ([1]) and $\{n = 2^m - 2 : m = 2, 3, \dots\}$ ([2] and independently [3]). Known are also the exact values of $\beta(n)$ for some $n \leq 63$ ([1], [4–7]). We recommend [8] as a survey of results on classical broadcasting and related problems.

New models.

In this paper we consider three new models of broadcasting, which we call "Messy broadcasting". We refer to them as M_1, M_2, and M_3.

In the classical broadcast model it is tacitly assumed that every node (member) of the scheme produces the broadcasting in the most clever way. For this it is assumed either, that there is a leader, who coordinates the actions of all members during the