1 Introduction

The general framework of a decision model is a set of situations together with a set of strategies belonging to these situations. In this paper we will only be interested in stationary strategies, which means independent of when we arrive at a certain situation we will always make the same decision. The aim is to minimize or maximize a certain cost function. The problem is that the outcome of certain decisions is random. In spite of this, we ask, if we can propose a certain subset of strategies as optimal with respect to the cost function.

For example, consider a mathematician having two different ways to prove a hypothesis e.g. say a direct proof and a proof by contradiction. After each hour of work he has to decide, whether to continue the work on the first or on the second type of proof.

Of course the mathematician does not know in advance, whether the next hour of work will be successful in the sense that he will be "closer" to the proof of the hypothesis afterwards. Anyhow, he will be looking for a fastest strategy to prove the hypothesis or to see that both approaches are unsuccessful.

More precisely we will consider random walks of several particles on a connected graph, that means each unit of time we are allowed to activate exactly one particle which, then describes one step of a random walk. Moreover, we are given a certain criterion concerning when the process stops.

The example with the mathematician can be described in terms of a 2 particles nearest-neighbor random walk on a line with two absorbing barriers. We ask for the fastest strategy to answer the question, whether at least one of the particles will be absorbed e.g. at the left barrier or both of them will be absorbed at the right barrier.

In the following we will also talk about best-case strategies when we mean fastest strategies and analogously worst-case strategies, when we mean slowest strategies. Note, that this terminology does not coincide with the "best-case" and "worst-case" terminology used in complexity theory.

Related questions have been studied from a more theoretical point of view for the meeting time of 2 particle random walks on arbitrary connected graphs in [Ald **91**], [CTW **92**] and [TW **93**], where an upper bound for the minimal running time has been established. Worst-case meeting times have been studied in [CTW **92**]. For a survey on random walks on graphs see the rich bibliography of Aldous [Ald **89**].

The paper is organized as follows:

In Section 2 we introduce the basic models we are going to work with and the central questions which are valid for the following sections.

Section 3 contains a best-case and worst-case analysis of the model with 2 particles and with two absorbing barriers.

Section 4 is one way to generalize Section 3 in the way that we now consider 3 particles on a line and ask for the fastest strategy to answer where the majority of the particles will be absorbed.

A second generalization of Section 3 is given in Section 5 where we are given s particles. We ask for the best-case strategy to answer the question, whether one of them will be absorbed at the left or all of them will be absorbed at the right barrier.

In Section 6 we compare the best-case and worst-case strategies for the models above.

Section 7 contains a further type of decision model with random walks. Instead of several particles on a line we consider the case of 3 and 4 particles on the n-circle C_n, where a collision of 2 particles leads to the end of the process.

In Section 8 we pose some open problems related to decision models with random walks.

2 A Basic Model

In Section 3 – 5 we are going to work with the following model:
We consider $s \geq 2$ particles on the non-negative integers $\{0, 1, \ldots, n\}$. Each unit of time we are allowed to knock exactly one of the particles which then describes a symmetric nearest-neighbor random walk. Let 0 and n be absorbing states of the random walk. It is well known (e.g. see [Chu **78**]) that if we only have one particle, say on state i, then the expected running time i.e. the number of knocks till absorption is equal to $i\,(n-i)$.

In Sections 3 – 5 we answer the following questions:

- *"How long does it take for $s = 2$ till either one particle will be absorbed at 0 or both will be absorbed at n ?"* (1)

  As a consequence of the law of large numbers one of these alternatives is reached in finite time almost surely. This means we are interested in the minimal and maximal running times.

- *"How long does it take for $s = 3$ till the majority of the particles will be absorbed in one of the states 0 or n ?"* (2)

  Again we are interested in extremal strategies.

- *"How long does it take for arbitrary $s \geq 3$ till either one particle will be absorbed at 0 or all of them will be absorbed at n ?"* (3)

If we label the positions of the particles with i and j for $s = 2$ and for convenience $0 \leq i \leq j \leq n$, the first question corresponds to the following equations:

$$F_n(i,j) = \begin{cases} 0\,, & \text{if} \quad i = 0\,, \\ i\,(n-i)\,, & \text{if} \quad j = n\,, \\ 1 + min\Big[\ \frac{1}{2}\big(F_n(i-1,j) + F_n(i+1,j)\big), & \\ \qquad\qquad \frac{1}{2}\big(F_n(i,j-1) + F_n(i,j+1)\big)\Big], & \text{if} \quad i < j\,, \\ 1 + \frac{1}{2}\big(F_n(i-1,i) + F_n(i,i+1)\big), & \text{if} \quad i = j\,. \end{cases} \tag{4}$$