REDUCTION OF REGULAR MULTIGRAPHS

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Abstract.

A multigraph is a graph with loops and multiple edges permitted. If all vertices of a multigraph have the same degree r, it is called r-regular.

The purpose of this paper is to give reduction procedures for the classes of r-regular multigraphs, where r is any natural number.

Moreover we present reduction procedures for the classes of r-regular multigraphs without loops.

A reduction procedure works as follows. Given an r-regular multigraph M, there is a sequence $M_1, \ldots, M_n$ of r-regular multigraphs s.t. $M = M_1$, and M_{i+1} can be obtained from M_i by one of finitely many wellspecified operations. M_n is the disjoint union of finitely many copies of multigraphs in a fixed finite set called the base.

We prove, that the list of operations we present is minimal, i.e. each operation is necessary to carry out the reduction procedure.

1. Reducing regular multigraphs

We shall present reduction procedures for r-regular multigraphs. Precisely, given any r-regular multigraph M, there is a sequence $M_1, \ldots, M_n$ of r-regular multigraphs s.t. $M_1 = M$ and M_{i+1} is obtained from M_i by deleting a vertex or a multiedge and by restoring regularity by defining new edges. M_n is the disjoint union of finitely many copies of multigraphs in a fixed finite set called the base, which we will specify depending on r.

A solution for simple r-regular graphs and a short history of the subject is given in [2].

This gives a natural characterisation of these classes of graphs by "prime elements" and generating rules. The bases we use have exactly one element, the "prime element" of the class, and all the other elements are generated from this element by the operations.

Furthermore, a reduction procedure for a class of graphs leads also to a device to give easy proofs by induction. Examples for such an approach are Johnson [3] and Statman [4].

Definition 1.1. *Let M be a multigraph with vertex set VM, having maximum degree less or equal to r. For $v \in VM$ the r-***deficiency of** *v is $d^r_M v := r - \deg_M v$, the* **maximum deficiency** *is $d^r M := \max\{d^r_M v | v \in VM\}$, and $s^r(M) := \sum_{v \in VM} d^r_M v$ is the r-***deficiency of** *M.*

First we consider r-regular graphs where r is even.