On Geometric Structures of dihedral Θ-Orbifolds

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Abstract

This article studies the geometric structures of dihedral orbifolds with a knot and a bridge as it's singular set and the three-dimensional sphere as the underlying space. These knotted graphs are connected with certain families of links factorized by their involution. In some of our cases the bridge is a tunnel of the knot. The greater part of the orbifolds described have a hyperbolic structure. Most of the fundamental groups appearing are two generator groups and moreover, they are generalized triangle groups, whose explicit representations are given.

1. Introduction

The purpose of the present article is to investigate geometrical structures of orbifolds where the singular set is a knot or a two-component link with a bridge and underlying space S^3. Examples of this kind have already been studied in the literature, so Helling, Mennicke and Vinberg [10] considered the trefoil knot with a bridge and determined the geometric structure of the orbifold with such a singular set for all branching indices k, l, m greater or equal to two (see figure 1).

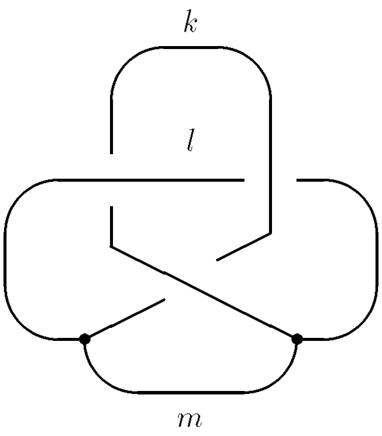

figure 1: *The trefoil with a bridge*

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Moreover the hyperbolic structures of the orbifolds with the Hopf-link or the 4_1^2-link with a bridge as singular set (see figure 2) were established by the first author [7].

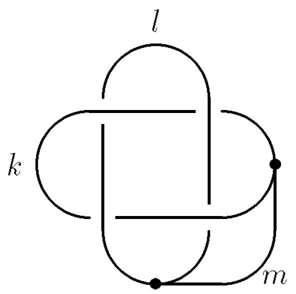

figure 2: *The double link 4_1^2 with a bridge*

In Theorems 1 and 2 we generalize these examples and construct families of hyperbolic dihedral orbifolds. Further examples of orbifolds similar to those above but having non-hyperbolic structure were obtained by Dunbar [6].

2. Embeddings of graphs in S^3 and orbifolds

It is well-known that if the singular set of a compact orbifold contains a graph (including also non-connected and multigraphs), then the order of any vertex is exactly three. Such a graph is called a cubic graph. Although cubic graphs are well-known as abstract graphs, they become rather complicate, as we are interested in their embeddings in S^3. Let us first consider two types of cubic multigraphs. It is obvious that there are only two abstract cubic graphs with two vertices (see figure 3).

figure 3: *Cubic graphs with two vertices and three edges*

We will call them **Θ-graph** and **"pince-nez"-graph** (as this name seems preferable to us than "handcuff"-graph the one chosen by Wolcott [14]). We remark that the singular set of the orbifolds from [10] (see figure 1) is isomorphic, as a graph, to the Θ-graph; and the singular set of the orbifold from [7] (see figure 2) is isomorphic, as a graph, to the "pince-nez"-graph.