m-Dependent Random Fields with Analytic Cumulant-Generating Function

F. Götze* L. Heinrich† C. Hipp‡

März 1994

Abstract

We consider m-dependent random fields of bounded random vectors and investigate the analyticity of the cumulant generating function of sums of these random vectors. Using the Kirkwood-Salsburg equations we derive upper bounds for the cumulant generating function and prove its analyticity in a neighbourhood of zero, where the normalized bounds and the neighbourhood are independent of the number of terms in the sum. The results are applied to spatial statistics in Poisson cluster processes and Boolean models and yield probabilities of large deviations as well as Berry-Esseen results for these statistics.

Keywords and phrases: m-dependent random field, cumulant-generating function, Kirkwood-Salsburg equations, Poisson cluster process, Boolean model. *1991 Mathematics Subject*

Classification: Primary 60 F 05 , 60 G 60 ;
Secondary 60 F 10, 60 G 55

Running title: Analyticity of the cumulant-generating function

1. Introduction and Summary

For j in the lattice $\mathbb{Z}_+^d \triangleq \{1, 2, \ldots\}^d$ let ξ_j be a random variable on a common probability space $[\Omega, \mathfrak{A}, \mathbb{P}]$ taking values in some measurable space $[\mathbb{X}_j, \mathfrak{X}_j]$. We shall assume throughout that the variables $\xi_j,\ j \in \mathbb{Z}_+^d$, are independent. For fixed integer $m \geq 0$ define

$$U_j \triangleq \times_{s=1}^d \{j_s, \ldots, j_s + m\} \quad \text{for} \quad j = (j_1, \ldots, j_d) \in \mathbb{Z}_+^d$$

*Universität Bielefeld, Fakultät für Mathematik, D-33501 Bielefeld, Germany
†TU Bergakademie Freiberg, Institut für Stochastik, Bernhard-von-Cotta-Str. 2, D-09596 Freiberg, Germany
‡Universität Karlsruhe, Lehrstuhl für Versicherungswissenschaft, Kronenstr. 34, D-76133 Karlsruhe, Germany

and

$$V_n^{(m)} \triangleq \times_{s=1}^d \{1,2,\ldots,n_s+m\} \quad \text{for} \quad n=(n_1,\ldots,n_d)\in \mathbb{Z}_+^d \quad , \quad V_n \triangleq V_n^{(0)}.$$

For $j \in \mathbb{Z}_+^d$ let f_j be $\mathbb{R}^p$-valued and measurable on the product space $\times_{i\in U_j}\mathbb{X}_i$; the mappings f_j are called *window functions* in [4]). We consider the $m-$dependent random field

$$X_j \triangleq f_j(\xi_i,\ i\in U_j), \quad j\in \mathbb{Z}_+^d \tag{1.1}$$

and the sum

$$S_n \triangleq \sum_{j\in V_n}(X_j - \mathbb{E}X_j) \tag{1.2}$$

It is the purpose of this paper to prove analyticity of the cumulant generating function (cgf)

$$\gamma_n(z) \triangleq \log \mathbb{E}\exp\{(z,S_n)\} \quad , \quad z\in \mathbb{C}^p \quad ,$$

in a neighbourhood $b(o,\delta)$ of the origin o, where δ is independent of n. This is done under the assumption that the random variables X_j are uniformly bounded,

$$\|X_j\| \le c \quad \text{for} \quad j\in \mathbb{Z}_+^d \quad . \tag{1.3}$$

Here and throughout we use the following notations for complex $p-$vectors $y,z\in\mathbb{C}^p$ and $r>0$:

$$(y,z) \triangleq \sum_{s=1}^p y_s\bar{z}_s$$

$$\|z\| \triangleq \sqrt{(z,z)}$$

$$b(y,r) \triangleq \{z\in\mathbb{C}^p : \|z-y\|\le r\}$$

(the same notation is used in Euclidean spaces and for any dimension >1) . The symbols $c, c_0, c_1,\ldots$ denote generic constants not depending on n. For finite sets V we write $|V|$ for the number of elements in V. We shall still use the same notation for the modulus of real or complex numbers.

In recent years results on Berry-Esseen bounds and Edgeworth expansions have been obtained for sums of the form (1.2). These results state that under additional assumptions on the field X_j (including conditional Cramér conditions) the remainder terms in these limit theorems have the same order as those in the corresponding theorems for independent X_j , see [3, 4, 5, 8, 9]. We mention that analogous estimates and expansions for arbitrary $d-$dimensional $m-$dependent random fields ($d\ge 2$) are still unknown or less precise in cases for which the representation (1.1) is not possible.

In the particular case of $m-$dependent sequences (i.e. $d=1$) the moment generating function of the sum admits a product representation in some neighbourhood of zero (see