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0. Introduction

Let $M_n = M_n(K)$ be the space of n by n matrices over an algebraically closed field K of characteristic 0. An important class of linear preserver problems deals with the investigation of linear transformations of the space M_n which leave invariant certain functions, subsets and/or relations on M_n. To be precise, let $\mathrm{End}(n^2)$ be the algebra of linear transformations of the vector space M_n, and $\mathrm{GL}(n^2)$ its group of invertible elements. If $X \subset M_n$, we define the *stabilizer monoids*

$$E_X = \{\sigma \in \mathrm{End}(n^2) : \sigma(X) \subset X\}$$

and

$$G_X = \mathrm{GL}(n^2) \cap E_X.$$

If θ is a binary relation on M_n, i.e., $\theta \subset M_n \times M_n$, one defines the monoids E_θ and G_θ similarly. For a function $f : M_n \to K$ the corresponding monoids are defined by :

$$E_f = \{\sigma \in E(n^2) : f \circ \sigma = f\}$$

and

$$G_f = \mathrm{GL}(n^2) \cap E_f.$$

One usually refers to the elements of E_X (E_θ, E_f) and G_X (G_θ, G_f) as the *linear preservers* of and the *nonsingular linear preservers*, respectively, of X (θ, f). It is often true that G_X, G_θ, or G_f is a subgroup of $\mathrm{GL}(n^2)$, and in that case we shall refer to it as the *stabilizer subgroup* of X, θ, or f. This is the case if X, θ are closed or open in the Zariski topology, or if f is a polynomial function.