Example 4 In Subsection 3.2 we proved that $\tilde{R}^{(1)} - \tilde{R}^{(n)} \geq \frac{1}{2} - \epsilon$. For an element a of order 4 in the multiplicative group of F, we (probabilistically) constructed a symmetric set $K \subseteq F$ such that:

(a) G's independence number was $\leq (1+o(1))16\log^2 v$.

(b) For all nonzero x, $\quad x \in K$ iff $ax \notin K$.

Condition (a) implies (1) above. Condition (b) appears slightly different from (2). We used it to infer that G is self complementary, hence $\chi(G^2) \leq v$ and $\tilde{R}^{(2)} \leq \frac{1}{2}$. However, Condition (b) has a simple K-code interpretation. Since $0 \neq x \in K$ implies $ax \notin K$, the set $\{(x, ax) : x \in F\}$ is a linear K-code. It has length 2 and dimension 1, hence $\tilde{R}^{(2)} \leq \frac{1}{2}$. $\square$

We have not found a symmetric set K satisfying Conditions (1) and (2) for $\alpha > 1/2$, and the problem of deciding if such a set exists remains open.

Acknowledgement

We thank Jim Roche for helpful discussions.

References

[1] P.K. Agarwal, N. Alon, B. Aronov, and S. Suri. Can visibility graphs be represented compactly? In *Proceedings of the 9th ACM Symposium on Computational Geometry*, pages 338–347, 1993.

[2] N. Alon. Monochromatic directed walks in arc colored directed graphs. *Acta Math. Acad. Sci. Hungar.*, 49:163–167, 1987.

[3] N. Alon, H. Lefmann, and V. Rődl. On an anti-ramsey type result. In *Colloq. Math. Soc. János Bolyai 60; Sets, Graphs and Numbers*, pages 9–22. Budapest, Hungary, 1991.

[4] P. Erdős. Some remarks on the theory of graphs. *Bulletin of the American Mathematical Society*, 53:292–294, 1947.

[5] T. Feder, E. Kushilevitz, and M. Naor. Amortized communication complexity. In *Proceedings of the 32nd Annual Symposium on Foundations of Computer Science*, pages 239–248, 1991.

[6] M.J. Ferguson and D.W. Bailey. Zero-error coding for correlated sources. Unpublished, 1975.

[7] R.L. Graham, B.L. Rothschild, and J.H. Spencer. *Ramsey Theory (Second Edition)*. Wiley Interscience, 1990.

[8] W. Haemers. On some problems of Lovász concerning the Shannon capacity of a graph. *IEEE Transactions on Information Theory*, 25(2):231–232, March 1979.

[9] M. Karchmer, R. Raz, and A. Wigderson. On proving super-logarithmic depth lower bounds via the direct sum in communication complexity. In *Proceedings of the 6th IEEE Symposium on Structure in Complexity Theory*, pages 299–304, 1991.

[10] N. Linial and U. Vazirani. Graph products and chromatic numbers. In *Proceedings of the 30th Annual Symposium on Foundations of Computer Science*, pages 124–128, 1989.

[11] L. Lovász. On the Shannon capacity of a graph. *IEEE Transactions on Information Theory*, 25(1):1–7, January 1979.

[12] M. Naor, A. Orlitsky, and P. Shor. Three results on interactive communication. *IEEE Transactions on Information Theory*, 39(5):1608–1615, September 1993.

[13] A. Orlitsky. Worst-case interactive communication I: Two messages are almost optimal. *IEEE Transactions on Information Theory*, 36(5):1111–1126, September 1990.

[14] F. P. Ramsey. On a problem of formal logic. *Proceedings of the London Mathematical Society*, 30(2):264–286, 1929.

[15] C. E. Shannon. The zero-error capacity of a noisy channel. *IRE Transactions on Information Theory*, 2(3):8–19, 1956.

[16] E. Sperner. Ein satz über untermengen einer endlichen menge. *Math. Z.*, 27:544–548, 1928.

[17] H. Witsenhausen. The zero-error side information problem and chromatic numbers. *IEEE Transactions on Information Theory*, 22(5):592–593, September 1976.

[18] A.C. Yao. Some complexity questions related to distributive computing. In *Proceedings of the 11th Annual ACM Symposium on Theory of Computing*, pages 209–213, 1979.