CHEVALLEY GROUPS OVER COMMUTATIVE RINGS
I. ELEMENTARY CALCULATIONS

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Abstract. This is the first in a series of papers dedicated to the structure of Chevalley groups over commutative rings. The goal of this series is to systematically develop methods of calculations in Chevalley groups over rings, based on the use of their minimal modules. As an application we give new direct proofs for normality of the elementary subgroup, description of normal subgroups and similar results due to E.Abe, G.Taddei, L.N.Vaserstein and others, as well as some generalizations. In this first part we outline the whole project, reproduce construction of Chevalley groups and their elementary subgroups, recall familiar facts about the elementary calculations in these groups, and fix a specific choice of the structure constants.

Introduction

The purpose of this series of papers is to systematically develop methods of calculations in Chevalley groups over commutative rings, based on the use of their minimal modules. Namely, we describe in **all** details two new approaches in the study of (exceptional) Chevalley groups over rings, sketched in [VPS] and [Va1]. The approaches are complementary, or, to put it plainly, antagonistic. The leading idea of one of them – what we call the **geometry of exceptional groups** – is that we can calculate with matrices for the exceptional groups as well. The leading idea of another one – the **decomposition of unipotents** – is that one can *completely* eliminate matrices from all usual calculations pertaining to the classical groups, considering only elementary matrices and isolated columns or rows instead. As is classically known, all latter calculations can be easily performed also in the exceptional groups [Sb2], [Ca2], [M], [S5].

As an application of these methods in the final paper of the series we give new direct proofs of the main structure theorems: normality of the elementary subgroups, classification of normal subgroups and the like. Of course for the classical cases these results are well-known (by J.S.Wilson, I.Z.Golubchik, A.A.Suslin, V.I.Kopeiko and many others), while for the exceptional ones have been obtained by E.Abe, G.Taddei and L.N.Vaserstein [Ta3], [V6], [A5] – [A7] fairly recently. Our method of proof is more straightforward and elementary and completely eliminates distinction between exceptional and classical cases. Moreover, even for classical groups, proofs based on it, are often much easier than the known ones.