ON THE RATE OF NORMAL APPROXIMATION IN $D[0,1]$

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Abstract.
The paper gives estimates of the rate of convergence in the Central Limit Theorems for stochastically continuous cadlag processes proved recently by Bézandry and Fernique (*Ann. Inst H. Poincare*, **28**) and Bloznelis and Paulauskas (to appear in *Stoch. Proc. Appl.*).

1. Introduction

Let $X = \{X(t),\ t \in [0,1]\}$ be a centered stochastically continuous random process with cadlag (right continuous and having left limits) sample paths. Let $D[0,1]$ denote the Skorohod space of cadlag functions on $[0,1]$ endowed with the usual topology. The process X is said to satisfy the Central Limit Theorem (CLT) in $D[0,1]$ if the distributions $\mathcal{L}(S_n)$ of the normalized sums $S_n = n^{-1/2}(X_1 + \cdots + X_n)$ converge weakly to the distribution of a Gaussian process, say Y, with sample paths in $D[0,1]$. Here $X_1, X_2, ...$ are independent copies of X. The aim of the paper is to estimate the rate of convergence in the following limit theorem proved recently by Bloznelis and Paulauskas (1993 b) and Fernique (1993).

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Theorem 1. *Let* $p,\ q \geq 2$. *Let* $f,\ g$ *be nonnegative increasing functions on* $[0,+\infty)$. *Let* X *be a r. process with mean* 0, *finite second moments, and sample paths in* $D[0,1]$ *satisfying*

$$\mathbf{E}\big(|X(s)-X(t)| \wedge |X(t)-X(u)|\big)^p \leq f(u-s), \tag{1.1}$$

$$\mathbf{E}|X(s)-X(t)|^q \leq g(t-s), \tag{1.2}$$

for all $s \leq t \leq u$, *and*

$$\int_0 f^{1/p}(u)\cdot u^{-1-1/p}du < \infty, \quad \int_0 g^{1/q}(u)\cdot u^{-1-1/(2q)}du < \infty. \tag{1.3}$$

Then the process X *satisfies the CLT in* $D[0,1]$.

We write $x\wedge y = min\{x;y\}$. For $x \in D[0,1]$ we shall denote $\|x\| = \sup_{t\in[0,1]}|x(t)|$. In the paper we estimate the uniform (Kolmogorov) distance between the distributions $\mathcal{L}(\|S_n\|)$ and $\mathcal{L}(\|Y\|)$.

Theorem 2. *Let conditions of Theorem 1 hold with the functions* $f(u) = c\cdot u^{1+\alpha}$ *and* $g(u) = c\cdot u^{1/2+\alpha}$, $\alpha > 0$. *Then there exists a constant* $C_1 = C_1(\mathcal{L}(Y),p,q,\alpha)$ *such that*

$$\sup_{r\geq 0}|\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)| \leq C_1(1+\mathbf{E}\|X\|^3)n^{-1/6}\log^{2/3}(n). \tag{1.4}$$

Here and in what follows letters C and c with indices or without denote constants which depend only on the quantities indicated in the brackets. Theorem 2 is a particular case of more general results formulated in section 2 below. Denote

$$\varDelta(r,S_n,Y) = |\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)|.$$

Theorem 2 gives the bound for $\varDelta(r,S_n,Y)$ which is uniform with respect to $r \geq 0$. Next theorem gives the so called non – uniform bound.

Theorem 3. *Let* $p > 3$ *and* $q \geq 2$. *Let conditions of Theorem 1 hold with the functions* $f(u) = cu^{1+\alpha}$, $g(u) = cu^{1/2+\alpha}$, $\alpha > 0$. *Assume that* $\mathbf{E}\|X\|^\beta < \infty$ *for some* $\beta > 3$. *Then there exists a constant* $C_2 = C_2(\mathcal{L}(Y),p,q,\alpha,\beta)$ *such that for* $r \geq 0$

$$|\mathbf{P}(\|S_n\| \leq r) - \mathbf{P}(\|Y\| \leq r)|(1+r^3) \leq C_2(1+\mathbf{E}\|X\|^\beta)n^{-1/6}\log^2(n). \tag{1.5}$$

Remarks. Let F and G be increasing continuous functions defined on $[0,1]$. Theorems $1,2$ and 3 remain true if one replaces $f(u-s)$ and $g(t-s)$ in the right hand sides of (1.1) and (1.2) by $f(F(u)-F(s))$ and $g(G(t)-G(s))$ respectively.

The rate of order $\mathcal{O}(n^{-1/6})$ appears usually when one considers the CLT for a Banach function space valued random elements, see, e.g., Paulauskas and Račkauskas (1989) and Bentkus et al (1990), moreover this rate is optimal in a sense, see ibidem. Earlier, the rate of convergence in the CLT in $D[0,1]$ was considered by Paulauskas and Juknevičiene (1989) and Paulauskas and Stieve (1990).