BOUNDS FOR CHARACTERISTIC FUNCTIONS OF POWERS OF ASYMPTOTICALLY NORMAL VARIABLES

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Abstract. We prove a partial extension of Vinogradovs estimates of trigonometric sums to the case of random variables. These generalizations are useful in limit theorems in probability theory and mathematical statistics.

1. Introduction and main results

The results in this paper extend the results in Götze et al. (1994) where upper bounds for $|\mathbb{E}\exp\{itf(S_n)\}|$ for polynomials f and normed sums S_n of i. i. d. random vectors $X_1, \ldots, X_n$ were proved assuming that the distribution of the summands had either a discrete or an absolutely continuous component without requiring moment conditions.

Under those assumptions it was possible to apply Vinogradov's estimates of trigonometric sums directly to a reduction of the original problem to a sum of Bernoulli random variables, respectively, to a sum of random variables which admit a Lebesgue density.

In this paper we will prove Vinogradov type estimates for the characteristic function of the kth power of sums of i. i. d. random variables with 'regular' distribution. This means that the random variables have a positive variance and a finite third absolute moment.

More generally the estimates hold for c. f. of the kth power of random variables S_n which are sums of two independent r. v. U_n and V_n which satisfy local upper concentration inequalities compared with Lebesgue measure and some uniformity condition. (see the Remarks following Theorem 1).

¿From now on we denote by $X_1, X_2, \ldots$ a sequence of i. i. d. real random variables with $\beta = \mathbb{E}|X_1|^3 < \infty$.

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Define $\sigma^2 = DX_1$ and

$$S_n = n^{-1/2}\sigma^{-1}\sum_{i=1}^{n}(X_i - \mathbb{E}X_1).$$

Let m, k be integers, $k > 1$, $m > 1$.

Let $B > 0$ and $\overline{x} \triangleq (x_1, \ldots, x_m, x'_1, \ldots, x'_m)$. We define a set in $\mathbb{R}^{2m}$ by the formula

$$\begin{aligned} &A(m,k,\overline{x},B) \\ &\triangleq \left\{ \overline{x} \in \mathbb{R}^{2m} : -0.5 < \sum_{j=1}^{m}(x_j^i - (x'_j)^i) \le 0.5, \right. \\ &\left. -B < x_i \le B, -B < x'_i \le B, i = \overline{1, \min(m;k)} \right\}. \end{aligned}$$

We say that a random variable X satisfies the condition $CF(k)$ if for all $m = 1, 2, \ldots k, 2k, 3k, \ldots$ and for independent copies $X_1, \ldots, X_m, X_1^1, \ldots, X_m^1$ of X there exists a constant $c(k,m)$ depending only on k and m such that for every $B > 0$ and all $0 < a \le B^2$ we have

$$\begin{aligned} &\mathbb{P}(\mathcal{A}(m,k,\bar{X}_a,B)) \\ &\le c(k,m)\sup_{|L|\le B}\mathbb{P}(\mathcal{A}(m,k,\bar{X}_L,B)), \end{aligned}$$

where $\bar{X}_a$ is a “shifted” random vector $\bar{X}$, that is

$$\bar{X}_a \stackrel{d}{=} (X_1 - a, \ldots, X_m - a, X'_1 - a, \ldots, X'_m - a).$$

Theorem 1. Let $X_1, X_2, \ldots$ be i. i. d. random variables with $\beta = \mathbb{E}|X_1|^3 < \infty$. Then for any positive integer k there exist absolute constants c_1, c_2, c_3 and a constant c_4 depending on β such that for all $n \ge 1$

$$\begin{aligned} &|\mathbb{E}\exp\{itS_n^k\}| \\ &\le c_4 n^{-c_1/(k^3 ln(k+1))}(1 + \sqrt{n}/|t|)^{c_2/(k^2 ln(k+1))} \end{aligned} \tag{0-1}$$

provided that

$$0 < |t| < c_3 n^{(k-0.5)/2} \tag{0-2}$$

and $S_n(\sqrt{n})$ satisfies the condition $CF(k)$, where

$$S_n(\sqrt{n}) = n^{-1/2}\sigma^{-1}\sum_{1}^{n}(X_i - \mathbb{E}X_i)\chi_{i,\sqrt{n}}$$

and $\chi_{i,u}$ denotes the indicator function of $\{|X_i - \mathbb{E}X_i| < \sigma u\}$.