ON A CONDENSED FORM FOR NORMAL MATRICES UNDER FINITE SEQUENCES OF UNITARY SIMILARITIES

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Summary

It is generally known that any Hermitian matrix can be reduced to a tridiagonal form by a finite sequence of elementary unitary similarities, namely Householder reflections. Recently A. Bunse–Gerstner and L. Elsner have found a condensed form to which any unitary matrix can be reduced, again by a finite sequence of Householder transformations. This condensed form can be considered as a pentadiagonal or block tridiagonal matrix with some additional zeros inside the band. Now the natural question arises, if there exists a condensed form for general normal matrices similar to some extent to the Bunse–Gerstner and Elsner form for the unitary case. In this paper we describe such a form (or, more precisely, a set of such forms) and show that the number of nonzero elements does not exceed $O(n^{3/2})$ where n is the order of the normal matrix given. We outline two approaches to constructing the condensed form. The first approach is a geometrical Lanczos–type one where we use the so–called generalized Krylov sequences. The second, more constructive approach, relies on a sequence of specially chosen Householder transformations.

Our condensed form can be thought of as a variable bandwidth form. It stays therefore invariant under Gaussian elimination without interchanges. We show that, for a normal matrix, this form is also invariant under the QR–algorithm.