Generalized Mehler semigroups and applications

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Abstract. We construct and study generalized Mehler semigroups $(p_t)_{t\geq 0}$ and their associated Markov processes $\mathbf{M}$. The construction methods for $(p_t)_{t\geq 0}$ are based on some new purely functional analytic results implying, in particular, that any strongly continuous semigroup on a Hilbert space H can be extended to some large Hilbert space E, with the embedding $H \subset E$ being Hilbert-Schmidt. The same analytic extension results are applied to construct strong solutions to stochastic differential equations of type $dX_t = C dW_t + AX_t dt$ (with possibly unbounded linear operators A and C on H) on a suitably chosen larger space E. For Gaussian generalized Mehler semigroups $(p_t)_{t\geq 0}$ with corresponding Markov process $\mathbf{M}$, the associated (non-symmetric) Dirichlet forms $(\mathcal{E}, D(\mathcal{E}))$ are explicitly calculated and a necessary and sufficient condition for path regularity of $\mathbf{M}$ in terms of $(\mathcal{E}, D(\mathcal{E}))$ is proved. Then, using Dirichlet form methods it is shown that $\mathbf{M}$ weakly solves the above stochastic differential equation if the state space E is chosen appropriately. Finally, we discuss the differences between these two methods yielding strong resp. weak solutions.

AMS Subject Classification (1991)
Primary: 47D07, 60H15
Secondary: 31C25, 60H10, 60G15

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