GIRSANOV–TYPE TRANSFORMATIONS OF LOCAL DIRICHLET FORMS: AN ANALYTIC APPROACH

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Abstract. Let $\mathbf{M}$ be a diffusion process with state space a Borel subset E of a Polish space. Suppose $\mathbf{M}$ is symmetric w.r.t. a σ–finite measure m, and the associated Dirichlet form $(\mathcal{E}, D(\mathcal{E}))$ on $L^2(E,m)$ admits a square field operator Γ. Let φ be an m–a.e. strictly positive function on E, that is supposed to be locally in $D(\mathcal{E})$ in a restricted sense. Under weak additional assumptions on $(\mathcal{E}, D(\mathcal{E}))$ it is shown analytically, that the bilinear form $\mathcal{E}^\varphi(u,v) = \int \Gamma(u,v)\,\varphi^2\,dm$ with domain $\mathcal{D}^\varphi = \{u \in D(\mathcal{E}); \int (\Gamma(u,u)+u^2)\,\varphi^2\,dm < \infty\}$ is closable on $L^2(E, \varphi^2 \cdot m)$, and the closure $(\mathcal{E}^\varphi, D(\mathcal{E}^\varphi))$ is a Dirichlet form associated with a diffusion $\mathbf{M}^\varphi$. $\mathbf{M}^\varphi$ is identified as the Girsanov–transform of $\mathbf{M}$ considered by M. Fukushima, Y. Oshima and M. Takeda. Moreover, a necessary and sufficient condition for a dense subspace in $D(\mathcal{E})$ to be dense in $D(\mathcal{E}^\varphi)$ is derived.
Applications include sufficient conditions for absolute continuity of diffusion processes w.r.t. Brownian motion on complete Riemannian manifolds (including a recent result of M. Takeda for Brownian motion in $\mathbb{R}^d$), reflected Brownian motions on smooth Euclidean domains, and diffusions associated with gradient Dirichlet forms on Banach spaces (extending results of S. Albeverio, M. Röckner and T.S. Zhang).

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0. Introduction

In recent years, several authors have investigated Girsanov transformations of symmetric diffusion processes, that preserve the strong Markov property as well as the symmetrizability, and the corresponding transformations of the associated analytic objects, in particular the Dirichlet forms, cf. [F 82], [MeZhe 84], [OT 87], [T 90], [ARZ 91], [T 93], [S 93], [Fi 93] and [FOT 94].

As an illustration, consider the following simple situation:

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Let $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t\geq 0}, (P_z)_{z\in\mathbb{R}^d})$ be the canonical Brownian motion on $\mathbb{R}^d$, $d \in \mathbb{N}$, and

$$\mathcal{E}(u,v) = \int \nabla u \cdot \nabla v \; dx, \quad u, v \in H^{1,2}(\mathbb{R}^d) \ ,$$

the associated Dirichlet form. Let φ be a function in $H^{1,2}_{\mathrm{loc}}(\mathbb{R}^d)$ satisfying $\varphi > 0$ a.e. and $\int |\nabla\varphi|^2 \, dx < \infty$. Fix a quasi–continuous modification $\tilde{\varphi}$ of φ, and let

$$M_t^{[\log \varphi]} := \int_0^t \frac{\nabla\tilde{\varphi}}{\tilde{\varphi}}(X_s) \, dX_s \ ,$$

defined up to the stopping time $\tau := \sup_{n\in\mathbb{N}} \inf \left\{ t > 0; \int_0^t \frac{|\nabla\tilde{\varphi}|^2}{\tilde{\varphi}^2}(X_s) \, ds \geq n \right\}$.