Fast Gossiping by Short Messages

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Abstract

Gossiping is the process of information diffusion in which each node of a network holds a packet that must be communicated to all other nodes in the network. We consider the problem of gossiping in communication networks under the restriction that communicating nodes can exchange up to a fixed number p of packets at each round. In the first part of the paper we study the extremal case $p = 1$ and we exactly determine the optimal number of communication rounds to perform gossiping for several classes of graphs, including Hamiltonian graphs and complete k-ary trees. For arbitrary graphs we give asymptotically matching upper and lower bounds. We also study the case of arbitrary p and we exactly determine the optimal number of communication rounds to perform gossiping under this hypothesis for complete graphs, hypercubes, rings, and paths.

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1 Introduction

Gossiping (also called total exchange or all–to–all communication) in distributed systems is the process of distribution of information known to each processor to every other processor of the system. This process of information dissemination is carried out by means of a sequence of message transmissions between adjacent nodes in the network.

The gossiping problem was originally introduced by the community of discrete mathematicians, to which it owes most of its terminology, as a combinatorial problem in graphs. Nonetheless, it was soon realized that, once cast in more realistic models of communication, gossiping is a fundamental primitive in distributed memory multiprocessor system. There are a number of situations in multiprocessor computation, such as global processor synchronization, where gossiping occurs. Moreover, the gossiping problem is implicit in a large class of parallel computation problems, such as linear system solving, Discrete Fourier Transform, and sorting, where both input and output data are required to be distributed across the network [5]. Due to the interesting theoretical questions it poses and its numerous practical applications, gossiping has been widely studied under various communication models. Hedetniemi, Hedetniemi and Liestman [9] provide a survey of the area. Two more recent surveys paper collecting the latest results are [7, 11]. The reader can also profitably see the book [14].

The great majority of the previous work on gossiping has considered the case in which the packets initially held by each node can be freely combined and the resulting messages transmitted in a constant amount of time, that is, that the time required to transmit a message is independent from its length. While this assumption is reasonable for short messages, it is clearly unrealistic in case the size of the messages becomes large. Notice that most of the gossiping protocols proposed in the literature require the transmission, in the last rounds of the execution of the protocol, of messages of size $\Theta(n)$, where n is the number of nodes in the network. Therefore, it would be interesting to have gossiping protocols that require only the transmission of bounded length messages between processors. In this paper we consider the problem of gossiping in communication networks under the restriction that communicating nodes can exchange up to a fixed number p of packets at each round.

1.1 The Model

Consider a communication network modeled by a graph $G = (V, E)$ where the node set V represents the set of processors of the network and E represents the set of the communication lines between processors.

Initially each node holds a packet that must be transmitted to any other node in the network by a sequence of *calls* between adjacent processors. During each call, communicating nodes can exchange up to p packets, where p is an *a priori* fixed integer. We assume that each processor can participate in at most one call at time. Therefore, we can see the gossiping process as a