PARTITIONS OF UNITY IN SOBOLEV SPACES OVER INFINITE DIMENSIONAL STATE SPACES

S.Albeverio
Fakultät für Mathematik
Ruhr-Universität-Bochum
D-44780 Bochum
Germany

Zhi-Ming Ma
Institute of Applied Mathematics
Academia Sinica
100080 Beijing
People's Republic of China

Michael Röckner
Fakultät für Mathematik
Universität Bielefeld
D-33615 Bielefeld
Germany

Abstract

We prove some general results on the existence of partitions of unity in Sobolev type spaces on various infinite dimensional manifolds. As special cases we obtain in particular, (continuous) partitions of unity a) in the Malliavin test functions on an abstract Wiener space; b) in the first order Sobolev space on pinned and free loop spaces; c) in the first order Sobolev spaces associated with reversible Fleming-Viot processes.

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0 Introduction

The purpose of this paper is to prove existence of (continuous) partitions of unity in Sobolev type spaces over infinite dimensional (not necessarily linear) state spaces. In the case of the Sobolev spaces D_p^r over an abstract Wiener space we even prove the existence of partitions of unity which are simultaneously in all of them, i.e., in the Malliavin test function space D^∞. This was, however, essentially known (see [Go71], [Pi77] and Remark 2.2 (ii)), but our general techniques developed in Section 1 below, leads to a quite simple new proof of this fact, which is presented in Section 2. Subsequently, we concentrate on partitions of unity in Sobolev type spaces of order one, more precisely Dirichlet spaces. If the Dirichlet space is regular, in particular its state space a locally compact separable metric space, the existence of partitions of unity is almost trivial (see Section 3 below). This is entirely different for more general Dirichlet forms on merely Polish state spaces. In Section 4 we consider a certain class of Dirichlet spaces $(\mathcal{E}, D(\mathcal{E}))$ on Polish state spaces studied in [RS93], and in Theorem 4.3 we give quite general conditions which ensure the existence of partitions of unity in $D(\mathcal{E})$. In Section 5 we present applications of Theorem 4.3. We prove that it applies, if e.g. the intrinsic metric associated with $(\mathcal{E}, D(\mathcal{E}))$ is uniformly equivalent to the initial metric (cf. Theorem 5.1.1). Furthermore, we show that partitions of unity exist for the following Dirichlet spaces which were constructed and analyzed recently (cf. [DR92], [ALR93], [MR92], [ORS93]); a) Dirichlet spaces on pinned and free loop spaces (cf. Theorem **??** and Remark 5.2.4); b) Dirichlet spaces on Banach spaces of gradient type (cf. Theorem 5.3.2); c) Dirichlet spaces on spaces of measures (cf. Theorem 5.4.1).

1 Preliminaries and some abstract results

For the convenience of the reader we first recall some basic concepts which will be used throughout this paper. Let E be a topological space. A covering of