On asynchronous Newton-Schulz type methods

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Abstract

The disadvantage of most known asynchronous iteration methods for linear and nonlinear equations seems to be, that the system to be solved, has to be related to an M-system. In this paper we will introduce an asynchronous method for nonlinear equations $F(x) = 0$, which is locally convergent, independent of the structures of the Jacobian of F . Admittedly, it seems easy to develop asynchronous variants of Newton's method, which converge even superlinearly, but the trouble of Newton's method cannot be avoided by asynchronous iterations, it is to solve again and again linear systems, which is on the whole a synchronized task. Sometimes it is also a hard task to compute the partial derivatives of F . Using a variation of the iterative method of Schulz (s. [9]) for inverting matrices, our method allows to solve nonlinear equations totally asynchronously, e.g. the iterations, the partial derivatives and the inverses of the Jacobians can be computed in parallel and without synchronization. Moreover, it converges locally superlinearly.

Key words: Asynchronous iterations, nonlinear equations, Newton's method, Schulz' method.

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1 Introduction

In [1] Baudet introduced asynchronous iteration methods for nonlinear fixed point problems and the more general *asynchronous iteration methods with memory*. In the latter case, there is a given operator

$$G : D_0^m \subset \mathbb{R}^{nm} \to D_0 \subset \mathbb{R}^n \tag{1}$$

for which a *fixed point* $\xi \in D_0$, e.g.

$$G(\xi, ..., \xi) = \xi, \tag{2}$$

is searched.

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