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ERGODICITY OF L^2–SEMIGROUPS AND EXTREMALITY OF GIBBS STATES

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Abstract. We extend classical results of Holley–Stroock on the characterization of extreme Gibbs states for the Ising model in terms of the irreducibility (resp. ergodicity) of the corresponding Glauber dynamics to the case of lattice systems with unbounded (linear) spin spaces. We first develop a general framework to discuss questions of this type using classical Dirichlet forms on infinite dimensional state spaces and their associated diffusions. We then describe concrete applications to lattice models with polynomial interactions (i.e., the discrete $P(\varphi)_d$–models of Euclidean quantum field theory). In addition, we prove the equivalence of extremality and shift–ergodicity for tempered Gibbs states of these models and also discuss this question in the general framework.

1. Introduction and preliminaries

Since the classical work of Holley and Stroock (cf. [HSt 76]) it is well–known that in the Ising model a Gibbs state μ is *extremal* if and only if the semigroup of operators given by the corresponding Glauber dynamics considered as an $L^2(\mu)$–semigroup is *irreducible* (or equivalently *ergodic*, see Section 2 below). The purpose of this paper is to prove this equivalence for classes of lattice models with non–compact, but linear spin space. The irreducibility of a symmetric $L^2(\mu)$–semigroup is equivalent to the irreducibility of the corresponding Dirichlet form, which in our cases are classical Dirichlet forms in the sense of [AR 90 a]. We thus also obtain a wide variety of examples of irreducible classical Dirichlet forms on infinite dimensional state space of which so far only a few (mainly Gaussian type) examples have been known.

To explain our main results more explicitly we need some preparations. We start with introducing the notion of Gibbs states associated with a "vector field" $b = (b_k)$.

Let E be a locally convex topological vector space (over the real numbers $\mathbb{R}$) which for simplicity is assumed to be Souslinean (i.e., the continuous image of a complete

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