Martingale Decomposition of Dirichlet Processes on the Banach space $C_0[0,1]$

T. J. Lyons[1]   M. Röckner[2]   T. S. Zhang[2*]

Abstract

We prove that for a given symmetric Dirichlet form of type $\mathcal{E}(u,v) = \int_E \langle A(z)\nabla u(z), \nabla v(z)\rangle_H \mu(dz)$ with $E = C_0[0,1]$ and $H =$ classical Cameron-Martin space the corresponding diffusion process (under P_μ) can be decomposed into a forward and a backward E-valued martingale. Applications to prove tightness of laws of diffusions of the above kind are given.

AMS Subject Classification
Primary: 60J60
Secondary: 60G44, 31C25, 60H10, 60B11
Key words: diffusions on Banach spaces, Dirichlet forms, Dirichlet processes, martingale decomposition
Running head: Infinite dimensional Dirichlet processes

[1]Department of Mathematics, Imperial College of Science, Technology and Medicine, Huxley Building, 180 Queen's Gate, London, U.K.

[2]Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, D-33501 Bielefeld, Germany

[*]on leave from HSH, Haugesund, Norway

1 Introduction and framework

Let $E = C_0[0,1]$ be the Banach space of all continuous functions on $[0,1]$ with initial value zero. E is endowed with the usual maximum norm denoted by $\| \quad \|$. Let μ be a probability measure on $(E, \mathcal{B}(E))$ with $\mathrm{supp}[\mu] = E$. Let $L^2(E;\mu)$ $(:= L^2(E \to \mathbb{R}; \mu))$ denote the corresponding L^2-space. Let H denote the classical Cameron-Martin space, i.e.,

$$H := \{h \in C_0[0,1] \mid h \text{ is absolutely continuous on } [0,1] \text{ and } \int_0^1 h'^2(s)ds < +\infty\}, \tag{1.1}$$

where ds denotes Lebesgue measure on $\mathbb{R}^1$. Then H is a Hilbert space with inner product

$$\langle h_1, h_2 \rangle_H := \int_0^1 h_1'(s) h_2'(s) ds.$$

Define the linear space

$$\begin{aligned} \mathcal{F}C_b^\infty \quad := \quad & \{u : E \to \mathbb{R} \mid u(z) = f(l_1, \ldots, l_m),\ m \in \mathbb{N}, \\ & f \in C_b^\infty(\mathbb{R}^m) \text{ and } l_1, \ldots, l_m \in E'\}, \end{aligned} \tag{1.2}$$

where $C_b^\infty(\mathbb{R}^m)$ is the set of all infinitely differentiable functions on $\mathbb{R}^m$ with all partial derivatives bounded and E' is the dual of E. Identifying H with its dual E' we have

$$E' \subset H \subset E \quad \text{densely and continuously,} \tag{1.3}$$

where E' is equipped with the usual operator norm. More precisely an element m in E', i.e., a signed measure of finite total variation on $\mathcal{B}([0,1])$, is identified with $\int_0^{\cdot} m([s,1])ds \in H$. Let $k \in E$ and define for $u \in \mathcal{F}C_b^\infty$

$$\frac{\partial u}{\partial k}(z) := \frac{d}{ds} u(z + sk)|_{s=0} \ , z \in E. \tag{1.4}$$

We denote by $\nabla u(z)$ the unique element in H representing the continuous linear map $k \mapsto \frac{\partial u}{\partial k} u(z), k \in H$.

Let $L^\infty(H)$ denote the collection of all continuous linear operators from H to H. Suppose we are given a strongly measurable map $A : E \to L^\infty(H)$, satisfying

(i) $A(z)$ is self-adjoint for each $z \in E$.