MULTIDIMENSIONAL VERSION OF THE RESULTS OF KOMLÓS, MAJOR AND TUSNÁDY FOR VECTORS WITH FINITE EXPONENTIAL MOMENTS

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Abstract. A multidimensional version of the results of Komlós, Major and Tusnády for the Gaussian approximation of the sequence of successive sums of independent random vectors with finite exponential moments is obtained.

0. Introduction

The statement of the problem is well known. It is required to construct on a probability space a sequence of independent random vectors $X_1, \dots, X_n$ and a corresponding sequence of independent Gaussian random vectors $Y_1, \dots, Y_n$ (this means that Y_i has the same mean and the same covariance operator as X_i, $i = 1, \dots, n$) so that the quantity

$$\Delta(X, Y) = \max_{1 \leqslant k \leqslant n} \left| \sum_{i=1}^{k} X_i - \sum_{i=1}^{k} Y_i \right| \tag{1}$$

would be so small as possible with large probability. Here

$$|x| = \max_{1 \leqslant j \leqslant d} |x_j|, \qquad x = (x_1, \dots, x_d) \in \mathbb{R}^d. \tag{2}$$

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Sometimes we shall use other notations for summands and, by analogy with [21], write

$$\Delta(\widetilde{X},\widetilde{Y}) = \max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^{k}\widetilde{X}_i - \sum_{i=1}^{k}\widetilde{Y}_i\right| \tag{3}$$

and so on.

In some sense this problem is one of the most important in probability approximations because many well-known probability theorems can be considered as consequences of results about strong approximation of sequences of sums by corresponding Gaussian sequences. This is related to the law of iterated logarithm, to several theorems about large deviations, to the estimates for the rate of convergence of the Prokhorov distance in the invariance principles [4, 15, 24] as well as to the Strassen-type approximations [26] (see, for example, [6]).

The rate of approximation in the invariance principle was investigated by many authors (see, e.g., [4, 15, 24] and the bibliography in [5, 6, 16–18]). Skorokhod [24] worked out a powerful method known now as Skorokhod embedding. For a long time it seemed that it is impossible to propose a method which could be better. However, in 1975, Komlós, Major and Tusnády (KMT) worked out a new method of dyadic approximation [10, 11]. With the help of this method they obtained an exhaustive solution of the problem mentioned above for sequences of independent identically distributed random variables. Sakhanenko [21] generalized and essentially improved KMT results for the case of non-identically distributed random variables.

The first attempts to extend the KMT approximations to the multidimensional case (see [2, 3, 7, 8, 14]) had a partial success only. Comparatively recently U. Einmahl [9] obtained multidimensional analogs of KMT results which are close to optimal. However, in his results (obtained in general case for the vectors with finite exponential moments) there is an unnecessary logarithmic factor. In this paper we improve the result of Einmahl [9] and obtain multivariate analogs of KMT results.

For formulations of results we need some notations. Writing $z\in\mathbb{R}^d$ (resp. $\mathbb{C}^d$), we always keep in mind that $z=(z_1,\dots,z_d)=z_1e_1+\dots+z_de_d$, where $z_j\in\mathbb{R}^1$ (resp. $\mathbb{C}^1$) and the e_j are the basis vectors. The scalar product of vectors $x,y\in\mathbb{R}^d$ (resp. $\mathbb{C}^d$) is denoted by $\langle x,y\rangle = x_1\overline{y}_1+\dots+x_d\overline{y}_d$. For $z\in\mathbb{R}^d$ (or $\mathbb{C}^d$) we shall use the Euclidean norm $\|z\|=\langle z,z\rangle^{1/2}$ and the maximum norm $|z|=\max\limits_{1\leqslant j\leqslant k}|z_j|$. For $b>0$ we denote $\log^+ b = \max\{1,\log b\}$. The distribution and the corresponding covariance operator of a random vector ξ will be denoted by $\mathcal{L}(\xi)$ and $\operatorname{cov}\xi$ (or $\operatorname{cov}F$, if $F=\mathcal{L}(\xi)$). The symbol I will be used for the identity operator. The formulas are numbered in each section separately. When we refer to the formula from another section, we indicate in addition the number of this section. For example, (1.2) is the formula (2) from Section 1.

In [30] the author has introduced a new class $\mathcal{A}_d(\tau)$ of d-dimensional probability distributions depending on a parameter $\tau\geqslant 0$. The class $\mathcal{A}_d(\tau)$ consists of d-dimen-