Gaussian Approximation of Local Empirical Processes Indexed by Functions

Uwe Einmahl*

Department of Mathematics, Indiana University, Bloomington, IN 47405, USA

David M. Mason**

Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA

Summary. An extended notion of a local empirical process indexed by functions is introduced, which includes kernel density and regression function estimators and the conditional empirical process as special cases. Under suitable regularity conditions a central limit theorem and a strong approximation by a sequence of Gaussian processes are established for such processes. A compact law of the iterated logarithm (LIL) is then inferred from the corresponding LIL for the approximating sequence of Gaussian processes.

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1. INTRODUCTION AND STATEMENTS OF MAIN RESULTS.

Deheuvels and Mason (1994a) introduced a notion of a local uniform empirical process indexed by a class of sets and proved a functional law of the iterated logarithm. Such local processes are very useful in the study of statistics which are functions of the observations in a suitable neighborhood of a point. For instance, Deheuvels and Mason (1994a) show how the pointwise Bahadur–Kiefer representation for the sample quantile and the law of the iterated logarithm for kernel density estimators follow readily from their results.

In this paper we extend the notion of the local empirical process to allow us to include kernel regression function estimators and the conditional empirical process within our setup. We then establish a weak convergence result and a strong invariance principle for such local processes. From our strong invariance principle we derive a general compact law of the iterated logarithm, which yields, among other results, the Deheuvels and Mason (1994a) functional law of iterated logarithm as a special case.

Let us begin by fixing some notation. Let $X, X_1, X_2, \ldots,$ be a sequence of i.i.d. $I\!R^d$ valued random vectors with distribution $I\!P$ on the Borel subsets $\mathcal{B}$ of $I\!R^d$. Given any $x \in I\!R^d$ and any measurable set $J \subseteq I\!R^d$, we set for any invertible bimeasurable transformation $h : I\!R^d \to I\!R^d$,

$$A(h) = x + hJ. \tag{1.1}$$

Let $\{h_n\}$ denote a sequence of invertible bimeasurable transformations from $I\!R^d$ to $I\!R^d$ and assume with $A_n = A(h_n)$ and $a_n = I\!P(A_n), n \geq 1$,