On the smallest maximal increment of partial sums of i.i.d. random variables

Uwe Einmahl[1*], **David M. Mason**[2*]

Summary

We study the almost sure limiting behavior of the smallest maximal increment of partial sums of n independent identically distributed random variables for a variety of increment sizes k_n, where k_n is a sequence of integers satisfying $1 \leq k_n \leq n$, and going to infinity at various rates. Our aim is to obtain universal results on such behavior under little or no assumptions on the underlying distribution function.

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[1] Department of Mathematics, Indiana University, Bloomington, IN 47405, USA
[2] Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA
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1. Introduction

Let $X, X_1, X_2, \ldots$ be a sequence of independent and identically distributed (i.i.d.) random variables with common nondegenerate distribution function F. For each integer $j \geq 1$, let $S_j = X_1 + \ldots + X_j$ and set $S_0 = 0$. We are interested in the limiting behavior of the *minimum* of the maximal increments of partial sums, where we consider different increment sizes. To be more specific, we shall study the asymptotic behavior of

$$m_n(k_n) = \min_{0 \leq i \leq n-k_n} \max_{0 \leq j \leq k_n} \mid S_{i+j} - S_i - j\ c_n \mid /a_n, \tag{1.1}$$

for suitable sequences of integers $1 \leq k_n \leq n$, where a_n and c_n are sequences of norming and centering constants depending on the underlying distribution function F and the sequence k_n. Our point of departure is the following result of M. Csörgő and Révész (1981). (For some clarification see Shao (1992) or page 112 of the monograph of Lin and Lu (1992.)

Let κ_n be a sequence of constants satisfying

(K.1) $1 \leq \kappa_n \leq n, n \geq 1$,
(K.2) $\kappa_n \nearrow \infty$, as $n \to \infty$,
(K.3) n/κ_n is nondecreasing,
(K.4) $\log n/\kappa_n \to 0$, as $n \to \infty$,
(K.5) $b_n := L\Big((nLn)\ /\kappa_n\Big)\ /\kappa_n$ is nonincreasing,

where we set $Lx = \log(x \vee e)$, $x \geq 0$.

As usual, denote by [x] the integer part of $-\infty < x < \infty$.

Theorem A (Csörgő and Révész (1981)) *Let X be a random variable with $EX = 0$ and Var $(X) = 1$, and let $k_n = [\kappa_n]$, where κ_n satisfies (K.1)-(K.5). Set $a_n = (8\ b_n)^{1/2}\ /\pi$ and $c_n = 0$ in (1.1).*
Then:

$$\liminf_{n\to\infty}\ m_n(k_n) = 1 \quad \text{a.s.} \tag{1.2}$$

Moreover, if $L(n/k_n)/LLn \to \infty$, then

$$\lim_{n\to\infty}\ m_n(k_n) = 1 \quad \text{a.s.} \tag{1.3}$$

One of the purposes of our paper is to obtain a version of Therore A without any condition at all on the underlying distribution. Among other results we shall obtain the following theorem, which is a consequence of more detailed results stated in the next section.

Theorem B *Let X be a nondegenerate random variable and let $k_n = [\kappa_n]$, where κ_n satisfies (K.1)-(K.5). There exist sequences of norming and centering constants a_n and c_n (depending on F and k_n) such that*

$$\liminf_{n\to\infty}\ m_n(k_n) \leq K_1 \quad \text{a.s.,} \tag{1.4}$$