The semi-simple zeta function of Shimura varieties associated to quaternion algebras

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Abstract

For the Shimura variety $Sh_{D,C}$ attached to a quaternion division algebra D over a totally real number field F and a sufficiently small open compact subgroup C of $D^{\times}(\mathbb{A}_f)$, and for every rational prime p which is unramified in F and such that $C = C_p \times C^p$ where C_p is a maximal compact subgroup of $D^{\times}(\mathbb{Q}_p)$ and C^p is a subgroup of $D^{\times}(\mathbb{A}_f^p)$, the semi-simple local zeta function $Z_p^{\mathrm{ss}}(Sh_{D,C}, s)$ is expressed as a product of integral powers of semi-simple local L–functions of automorphic representations of $D^{\times}(\mathbb{A})$. In an appendix a mistake occuring in the constructions of Langlands and Rapoport needed to state their conjecture on the reduction of a Shimura variety in the most general case is corrected. This conjecture is proved for $Sh_{D,C}$ up to a question concerning the reduction of abelian varieties of CM–type which is discussed in another appendix.

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