The martingale problem for pseudo-differential operators on infinite-dimensional spaces

V. Bogachev [1)] P. Lescot [2)] M. Röckner [3)]

Abstract

A martingale problem for pseudo-differential operators on infinite dimensional spaces is formulated and the existence of a solution is proved.

AMS Subject Classification
Primary: 60 G 44
Secondary: 47 G 30, 60 J 75
Key words: pseudo-differential operators in infinite dimensions, martingale problems, cadlag processes in nuclear spaces, tightness
Running head: Martingale problems in infinite dimensions

[1)] Department of Mechanics and Mathematics, Moscow State University, 119899 Moscow, Russia
[2)] Département de Mathématiques, Université de Paris VI, 4 Place Jussieu, 75252 Paris Cedex 05, France
[3)] Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany

0 Introduction

The purpose of this paper is to formulate and solve a martingale problem for pseudo-differential operators on infinite dimensional state space. We thus provide a routine machinery to construct (non-trivial) infinite dimensional processes which are merely càdlàg and have not been obtained before by other means. We emphasize, however, that this work is only on existence of solutions to these martingale problems, not on uniqueness. The uniqueness, which is known to be already extremely difficult in infinite dimensions if we are merely dealing with "differential" (i.e., local) operators, will be studied in a forthcoming paper.

The state spaces E treated here are duals to countably nuclear spaces and the definition of a pseudo-differential operator $p(\cdot, D)$ on E is taken from [Hr82, Hr87]. There are several possibilities to introduce pseudo-differential operators in infinite dimensions. One of them is based on the idea to use Fourier transforms of measures as test functions. The approach to Feynman integrals developed in [AH-K76, AH-K77] was based on this idea. Earlier in [Fom68] infinite-dimensional differential operators were considered in the framework of the duality between spaces of functions and spaces of measures (replacing the finite-dimensional duality between the Schwartz test function space S and its dual S'). This latter approach was then further developed by several authors. The most convenient version for us is that due to A. Yu. Hrennikov (cf. [Hr82] and also [Hr87] and the references therein). In particular, the domain of $p(\cdot, D)$ is hence a space of Fourier transforms of measures. Thus one can avoid the use of a reference measure, as Lebesgue measure is in finite dimensions (which does not exist on our state space E if $\dim E = \infty$). We refer to Sections 1 and 2 for precise details, but emphasize here that it definitely is a natural and direct way to extend the definition of a pseudo-differential operator to infinite dimensions. A further justification for this from a probabilistic point of view is given by the results of this paper.

The existence of the solution to the martingale problem for $p(\cdot, D)$ on E is then proved in Section 3. We thus extend fundamental work of D.W. Stroock (cf. [St75]) and, in particular, the more recent work by W. Hoh (cf. [H92, H94a]). The problems in the proof arise in part from the non-metrizability of E and hence of D_E (i.e., the space of càdlàg paths from