A Converse Coding Theorem for Mismatched Decoding at the Output of Binary-Input Memoryless Channels *

V.B.Balakirsky
The author is from The Data Security Association "Confident",
St.-Petersburg, Russia
E-mails:
vbal@stoic.spb.su
vbal@newton.spb.su
vbal@mathematik.uni-bielefeld.de

Abstract. An upper bound on the maximal transmission rate over binary-input memoryless channels, provided that the decoding decision rule is given, is derived. If the decision rule is equivalent to the maximum likelihood decoding (matched decoding), then the bound coincides with the channel capacity. Otherwise (mismatched decoding), it coincides with a known lower bound.

Key words :
channel capacity, mismatched decoding.

*The work was supported by a Scholarship from the Swedish Institute, Stockholm, Sweden.

1 Introduction

Shannon's coding theorem on the capacity of memoryless channels [1] may be presented as a result of maximization of the transmission rate over all possible block codes and all possible decoding algorithms. An open information theory problem is to generalize this theorem to the case when optimization over decoding algorithms is forbidden [2-10]. More precisely, the decoder calculates the value of some distortion (metric) function for each codeword and makes a decision that the codeword with the smallest distortion was sent. Since the distortion function is not necessarily matched to the channel's characteristics, the maximal transmission rate can be less than the channel capacity. The current state of the mismatched problem and its connections with other open problems of information and coding theory are given in the recent paper by I.Csiszár and P.Narayan [9], and we cannot do it better.

We deal with the binary-input memoryless channels and prove a converse statement to the direct coding theorem [3]. As a result, we obtain that the maximal transmission rate over any binary-input memoryless channel can be found for any distortion function using a single-letter characterization.

The paper is organized as follows. In Section 2 we introduce some notations, which will be used in the analysis. The main result is formulated and discussed in Section 3. In Section 4 we discuss the basic ideas of the proof of the theorem, which lead to the results, called a 'combinatorial approximation lemma' and a 'permutation lemma'. Section 5 is devoted to the proof of the combinatorial approximation lemma, and Section 6 is devoted to the proof of the permutation lemma. Some properties of the mismatched decoding and basic ideas of the proof are illustrated in the Appendix for specific data.

2 Notation

1. The channel's input and output alphabet will be denoted by X and Y, respectively. We assume that $X = \{0, 1\}$, but write X instead of $\{0, 1\}$ meaning that many of the further considerations can be extended to a general case.

2. The number of symbols $x \in X$ in $\mathbf{x} \in X^n$, the number of symbols $y \in Y$ in $\mathbf{y} \in Y^n$, and the number of pairs of symbols $(x, y) \in X \times Y$