CORRELATED SOURCES HELP TRANSMISSION OVER AN ARBITRARILY VARYING CHANNEL

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Abstract

It is well–known that the deterministic code capacity (for the average error probability criterion) of an arbitrarily varying channel (AVC) either equals its random code capacity or zero. Here it is shown that if two components of a correlated source are additionally available to the sender and receiver, respectively, the capacity always equals its random code capacity.

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1. Introduction and Result

A well–known dichotomy [1] in the behaviour of the deterministic code capacity (for the average error probability criterion) $\overline{C}(\mathcal{W})$ for an arbitrarily varying channel (AVC) with set of transmission matrices $\mathcal{W}$ is this: either $\overline{C}(\mathcal{W}) = 0$ or else $\overline{C}(\mathcal{W}) = C_R(\mathcal{W})$, the random code capacity $\min\limits_{W \in \overline{\mathcal{W}}} C(W)$, where $C(W)$ is the ordinary capacity of the discrete memoryless channel (DMC) W and $\overline{\mathcal{W}}$ is the convex closure of $\mathcal{W}$. It can happen that $C_R(\mathcal{W}) > 0$ and $\overline{C}(\mathcal{W}) = 0$.

We prove here that in the presence of a correlated source $(U^n, V^n)_{n=1}^\infty$ (which is independent of the message) with $I(U \wedge V) > 0$ and access of the sender to U^n and of the receiver to V^n this cannot happen!

Clearly, if $(U^n)_{n=1}^\infty$ and $(V^n)_{n=1}^\infty$ have positive common randomness, $CR(U,V)$ then the result is readily established by the elimination technique of [1]. Here the common randomness $CR(U,V)$ of a correlated source $(U^n, V^n)_{n=1}^\infty$ is defined as the maximal real R such that for all $\varepsilon > 0$ and sufficiently large n there exist functions $\phi : \mathcal{U}^n \to \{1, 2, \dots, 2^{n(R-\varepsilon)}\}$ and $\Phi : \mathcal{V}^n \to \{1, 2, \dots, 2^{n(R-\varepsilon)}\}$ with $\Pr\big(\phi(U^n) \neq \Phi(V^n)\big) < \varepsilon$ (c.f. [2]).

So the interesting case arises when

(a) the common randomness $CR(U,V)$ of the correlated source is 0

and

(b) $\overline{C}(\mathcal{W}) = 0$, but $C_R(\mathcal{W}) > 0$.

Now we know not only that a positive $CR(U,V)$ can help to make the channel capacity positive, but from [3] also that a positive channel capacity $\overline{C}(W)$ can be used to make the common randomness positive. However, we are confronted with the situation where *both quantities are* 0 !

Actually we can always find binary–valued functions f and g so that $\big(f(U_t), g(V_t)\big)_{t=1}^\infty$ satisfies $I\big(f(U) \wedge g(V)\big) > 0$, if $I(U \wedge V) > 0$.

So we can assume that we are given a binary correlated source with alphabets $\mathcal{U} = \mathcal{V} = \{0, 1\}$.

Quite surprisingly, we found a complete solution of the problem.

Theorem. *For an AVC $\mathcal{W}$ let the sender observe $U^n = U_1, \dots, U_n$ and let the receiver observe $V^n = V_1, \dots, V_n$, where $(U^n, V^n)_{n=1}^\infty$ is a memoryless correlated source (which is independent of the message) with generic pair of RV's (U,V) having mutual information $I(U \wedge V) > 0$. Then the capacity $\overline{C}\big(\mathcal{W}, (U,V)\big)$ for deterministic codes and the average error criterion equals the random capacity $C_R(\mathcal{W})$.*