Topological Classification of Complete Intersections *

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Abstract- We show that the homeomorphism type of the complete intersection $X_n(d_1, d_2, \cdots, d_r)$ is determined by its Pontryagin classes, Euler characteristic and the product $d_1 d_2 \cdots d_r$, if $d_1 d_2 \cdots d_r$ has no prime factors less than $\frac{n+3}{2}$. We also prove that, when n is even the homotopy type of $X_n(d_1, d_2, \cdots, d_r)$ is determined by its Euler charateristic, signature and the product $d_1 d_2 \cdots d_r$, in the case $d_1 d_2 \cdots d_r$ has no prime factors less than $\frac{n+3}{2}$. The latter confirms a conjecture of Libgober and Wood [8].

Sur les classifications topologiques des intersections complètes

Résumé- Nous montrons que le type d'homeomorphisme d'une intersection complète $X_n(d_1, d_2, \cdots, d_r)$ est déterminé par ses classes de Pontryagin, son nombre d'Euler, et le product $d_1 d_2 \cdots d_r$ dans le cas que $d_1 d_2 \cdots d_r$ n'a pas des facteurs primes moins de $\frac{n+3}{2}$. Nous montrons aussi que quand n est paire, le type d'homotopie de $X_n(d_1, d_2, \cdots, d_r)$ est déterminé par son nombre d'Euler, des mêmes signatures et le product $d_1 d_2 \cdots d_r$ dans le cas $d_1 d_2 \cdots d_r$ n'a pas des facteurs primes moins de $\frac{n+3}{2}$. Ce dernier résultat confirme une conjecture de Libgober et Wood [8].

Version française abrégée. Soit $X_n(\mathbf{d}) \subset CP^{n+r}$ une intersection complète, obtenne comme l'intersection transversale des r hypersurfaces dans CP^{n+r} définées par des polynomes de degré $\mathbf{d} = (d_1, d_2, \cdots, d_r)$. On appelle $d_1 \cdots d_r$ le degré total de $X_n(\mathbf{d})$.

Dans leur article [8], Libgober et Wood montrent que pour chaque mombre entier k, il y a k intersections complètes diffeomorphiques avec des multidegrés différents dans toutes dimensions.

Dans cette Note, nous annoncons quelques résultats sur le type d'homeomorphism et d'homotopie des intersections complètes.

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Comme examples, on a

Théorèm 0.1 *Soient $X_n(\mathbf{d})$ et $X_n(\mathbf{d}')$ deux intersections complètes avec des mêmes degrés totals d. Supposons que d n'a pas des facteurs primes moins de $\frac{n+3}{2}$(i.e., Si $p|d$, alors, $2p \geq n+3$). Alors, $X_n(\mathbf{d})$ et $X_n(\mathbf{d}')$ sont homeomorphiques si et seulement si ils ont des mêmes classes de Pontryagin et des mêmes nombres d'Euler.*

Théorèm 0.2 *Soit n est un entier pair. Soient $X_n(\mathbf{d})$ et $X_n(\mathbf{d}')$ deux intersections complètes avec des mêmes degrés totals d. Supposons que d n'a pas des facteurs primes moins de $\frac{n+3}{2}$(i.e., Si $p|d$, alors, $2p \geq n+3$). Alors, $X_n(\mathbf{d})$ et $X_n(\mathbf{d}')$ sont equivalents homotopiques si et seulement si ils ont des mêmes nombres d'Euler et des mêmes signatures.*

On note que, théorème 0.2 confirme une conjecture de Libgober et Wood [8].

The purpose of this note is to announce several theorems about the homeomorphism and homotopy classification of complex complete intersections and to sketch out their proofs.

1. Nonsingular complete intersections- Let $X_n(\mathbf{d}) \subset CP^{n+r}$ denote a complete intersection, the transversal intersection of r hypersurfaces in CP^{n+r} defined by r homogeneous polynomials of multidegree $\mathbf{d} = (d_1, d_2, \cdots, d_r)$, we call the product $d_1 \cdots d_r$ the *total degree* and denote it by d. It is an easy fact that the total degree is a homotopy invariant of the complete intersection $X_n(\mathbf{d})$ if the dimension $n \geq 3$. Up to dimension $(n-1)$, the homolgy of a complete intersection can be read out from Lefschetz's hyperplane Theorem. It is a classical observation of R.Thom that the diffeomorphism type of a complete intersection is determined by the dimension n and its multidegree $\mathbf{d}$. In [8], Libgober-Wood proved that, for any integer k, there are k diffeomorphic complete intersections with different multidegrees in every dimension. It is a natural problem to ask when two complete intersections are diffeomorphic, homeomorphic and/or homotopy equivalent.

In the complex dimensions $n = 1$ and 3, the diffeomorphism classification of the complete intersections is known as a corollary of surface theory and the general classification of simply connected 6 manifolds [11]. For dimension $n = 2$, the topological and homotopy classification is well known by Freedman's work[3]. In dimension 4, the topological classification was completed in my joint paper with S.Klaus [2]. For higher dimensions, it seems very hard to obtain a complete answer about the classification problem of complete intersections up to homotopy or homeomorphism.

The first general results occured in the early 80's by a series works of Libgober and Wood. In [6], they proved that, when n is odd, the homotopy type of $X_n(\mathbf{d})$ depends only on its total degree and Euler charateristic provided d has no prime factors less than