LARGE DEVIATIONS FOR PARTIAL SUMS U-PROCESSES

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Dedicated to Hans Zessin on the occasion of his 50th birthday

ABSTRACT. The large deviations principle (LDP) is known to hold for U-statistics of real valued kernel functions of Polish space valued i. i. d. random variables, where moreover appropriate exponential tail conditions are assumed to hold. We prove that similar conditions suffice for the large deviations principle to carry over to the partial sums U-processes corresponding to the statistics.

§1 INTRODUCTION

For a sequence of $\mathbb{R}^d$-valued i. i. d. random variables X_i and with a law of X_i having a finite moment generating function, Varadhan [19] proved a LDP for the partial sums process

$$S_n(t) = \frac{1}{n} \sum_{i=1}^{[nt]} X_i, \quad t \in [0,1].$$

Mogulskii [16] extended the result to other scalings and also accomodated moment generating functions, which are finite only in a neighbourhood of the origin. In [15] the case of X_i which are real valued functions of a regular finite state Markov chain is treated. Schuette [18] analysed real valued, independent but not necessarily identical distributed X_i under more restrictive conditions on the corresponding moment generating functions. Moreover Dembo and Zajic [4] proved an extension of Varadhan's result to cover the LDP for

$$L_n(t) = \frac{1}{n} \sum_{i=1}^{[nt]} \delta_{X_i}, \quad t \in [0,1], \tag{1.1}$$

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or for the Banach space valued partial sums process $S_n(\cdot)$, where the X_i fulfill Markov dependence or mixing conditions. They observed that the special conditions, which guarantees the LDP to hold for the empirical measures of Polish space valued random variables and for the empirical means of Banach space valued random variables, suffice for the LDP to carry over to the corresponding partial sums processes.

In this paper we investigate the LDP for partial sums U-processes, which means partial sums of the following form:

$$U_n(t) = \frac{1}{\binom{n}{m}} \sum_{C_m^{[nt]}} h(X_{i_1}, \ldots, X_{i_m}), \quad t \in [0,1]. \tag{1.2}$$

Here the X_i are i. i. d. random variables with values in a Polish space S, and h is a measurable, symmetric $\mathbb{R}^d$-valued function, called kernel function, where symmetric means that h is invariant under all permutations of its arguments. C_m^k with $k, m \in \mathbb{N}$ denotes the set $\{(i_1, \ldots, i_m) : 1 \leq i_1 < \cdots < i_m \leq n\}$. To be more precise, we get an LDP in $L_\infty([0,1])$, the space of almost surely bounded, measurable $\mathbb{R}^d$-valued functions on $[0,1]$, where $[0,1]$ is equipped with Lebesgue measure unless explicitly stated otherwise. The LDP for U-statistics of degree $m \in \mathbb{N}$, which means statistics of the form

$$U_n = \frac{1}{\binom{n}{m}} \sum_{C_m^n} h(X_{i_1}, \ldots, X_{i_m}),$$

with h and C_m^n as above, is proved for $\mathbb{R}^d$-valued h and Polish space valued X_i by the authors [9] in the i. i. d. case. An extended version is proved in [8], where the function h satisfies the condition

$$\int_{S^m} \exp(\Theta \|h(x_1, \ldots, x_m)\|) \, d\mu^m(x_1, \ldots, x_m) < \infty,$$

for each $\Theta > 0$, where μ denotes the common law of X_i. We will prove, that similar conditions suffice for the LDP to carry over to the partial sums U-processes defined as in (1.2).

While U-statistics were introduced by Hoeffding [10] as a generalization of the empirical mean to the case of multivariate functions, later different types of stochastic processes related with U-statistics have been studied, see for example Loynes [11],