ADDENDUM TO THE PAPER "DIRICHLET FORMS AND MARKOV PROCESSES: A GENERALIZED FRAMEWORK INCLUDING BOTH ELLIPTIC AND PARABOLIC CASES"

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Abstract. We extend the main result in [A/M/R], which is a complete characterization of all Dirichlet forms defined on some L^2–space $L^2(E,m)$ associated with m–tight special standard processes, to the framework of generalized Dirichlet forms.

0. Introduction

In our previous work [St] we introduced the class of *generalized Dirichlet forms* which contains symmetric and coercive Dirichlet forms as treated in [Fu] resp. [M/R] and time dependent Dirichlet forms as considered in [O], but also examples of an entirely new kind such as fractional diffusion operators and h–transformations (cf. [St] for a detailed discussion). The main result in [St] is the identification of an analytic property of *generalized Dirichlet forms* defined on $L^2(E,m)$, called *quasi–regularity* in analogy with [A/M/R] and [M/R], which allows the construction of an associated m–tight special standard process. This construction generalizes one half of the above mentioned result in [A/M/R] (namely the "sufficiency part"). In the present paper, we also carry over the "necessity part" of the result in [A/M/R] to our more general situation. More precisely: let $\mathcal{E}$ be a generalized (semi–) Dirichlet form on $L^2(E,m)$ and assume that there exists an m–tight special standard process $\mathbb{M}$ associated with $\mathcal{E}$. Then we shall prove in Theorem 1.6 that $\mathcal{E}$ is quasi–regular and, moreover, $\mathcal{E}$ is properly associated in the resolvent sense with $\mathbb{M}$ (cf. Definition 1.4 for the precise meaning).

The proof of the necessity of quasi–regularity of Dirichlet forms associated with m–tight special standard processes given in [A/M/R] and [M/R] cannot be carried over directly to the case of *generalized Dirichlet forms*. The main difference to the classical case lies in the fact that 1–excessive functions w.r.t. m–tight special standard processes associated with *generalized Dirichlet forms* are not quasi–continuous in general as seen for example in the case of the uniform motion on the real line.

1. Preliminaries and main results

Let us briefly recall the framework of generalized (semi–) Dirichlet forms: Let E be a topological Hausdorff space whose Borel–σ–algebra $\mathcal{B}(E)$ is generated by the set $C(E)$ of all continuous functions on E and m a σ–finite measure on $(E, \mathcal{B}(E))$ such that $\mathcal{H} = L^2(E, \mathcal{B}(E), m)$ is separable. As usual denote with $\mathcal{B}_b(E)$ (resp. $\mathcal{B}_b^+(E)$) the set of all bounded (resp. bounded and non–negative) $\mathcal{B}(E)$–measurable functions on E.

Let $(\mathcal{A}, \mathcal{V})$ be a real valued coercive closed form on $\mathcal{H}$ with sector constant $K \geq 1$. Note that $\mathcal{V}$ equipped with inner product $\tilde{\mathcal{A}}_1(\cdot,\cdot)$ (where $\tilde{\mathcal{A}}_\alpha(u,v) := 1/2(\mathcal{A}(u,v) + \mathcal{A}(v,u)) + \alpha(u,v)_\mathcal{H}$ for $u, v \in \mathcal{V}, \alpha > 0$) is again a separable real Hilbert space. Denote with $\|\cdot\|_\mathcal{V}$ the corresponding norm.

Identifying $\mathcal{H}$ with its dual $\mathcal{H}'$, we obtain that $\mathcal{V} \subset \mathcal{H} \subset \mathcal{V}'$ densely and continuously. Denote by $\langle \cdot, \cdot \rangle$ the dualization between $\mathcal{V}$ and $\mathcal{V}'$.

Let Λ with domain $D(\Lambda, \mathcal{V}')$ be a linear operator on $\mathcal{V}'$ satisfying the following conditions:

D1 (i) $(\Lambda, D(\Lambda, \mathcal{V}'))$ generates a strongly continuous semigroup $(U_t)_{t\geq 0}$ of bounded linear operators on $\mathcal{V}'$.

(ii) $(U_t)_{t\geq 0}$ can be restricted both to a strongly continuous *contraction* semigroup on $\mathcal{H}$ and to a strongly continuous semigroup of bounded linear operators on $\mathcal{V}$.

Denote with $(\Lambda, D(\Lambda, \mathcal{H}))$ and $(\Lambda, D(\Lambda, \mathcal{V}))$ the generators corresponding to the restricted semigroups and with $(\widehat{\Lambda}, D(\widehat{\Lambda}, \mathcal{V}'))$ the dual operator of $(\Lambda, D(\Lambda, \mathcal{V}))$. Define the Hilbert spaces

$$\mathcal{F} := D(\Lambda, \mathcal{V}') \cap \mathcal{V} \text{ with norm } \|u\|_\mathcal{F}^2 := \|u\|_\mathcal{V}^2 + \|\Lambda u\|_{\mathcal{V}'}^2, \text{ and}$$
$$\widehat{\mathcal{F}} := D(\hat{\Lambda}, \mathcal{V}') \cap \mathcal{V} \text{ with norm } \|u\|_{\widehat{\mathcal{F}}}^2 := \|u\|_\mathcal{V}^2 + \|\hat{\Lambda} u\|_{\mathcal{V}'}^2 \ .$$

Since $U_t : \mathcal{H} \to \mathcal{H}$ is a contraction for $t \geq 0$ it follows $(\Lambda u, u)_\mathcal{H} \leq 0$ for $u \in D(\Lambda, \mathcal{H})$. By the density of $D(\Lambda, \mathcal{V})$ in $\mathcal{F}$ (cf. [St, II.1]) we can find for each $u \in \mathcal{F}$ a sequence $(u_n)_{n\geq 1} \subset D(\Lambda, \mathcal{V})$ converging to u in $\mathcal{F}$. Consequently $\langle \Lambda u, u \rangle = \lim_{n\to\infty} \langle \Lambda u_n, u_n \rangle = \lim_{n\to\infty} (\Lambda u_n, u_n)_\mathcal{H} \leq 0$. Similarly we obtain $\langle \hat{\Lambda} u, u \rangle \leq 0$ for $u \in \widehat{\mathcal{F}}$.

Then we can define the form

$$\mathcal{E}(u,v) := \begin{cases} \mathcal{A}(u,v) - \langle \Lambda u, v \rangle & \text{if } u \in \mathcal{F}, v \in \mathcal{V} \\ \mathcal{A}(u,v) - \langle \hat{\Lambda} v, u \rangle & \text{if } u \in \mathcal{V}, v \in \widehat{\mathcal{F}}\,, \end{cases} \tag{1.1}$$

and $\mathcal{E}_\alpha(u,v) := \mathcal{E}(u,v) + \alpha(u,v)_\mathcal{H}$ for $\alpha > 0$ (cf. [St, II.2])). $\mathcal{E}$ is well–defined and positive definite (i.e. $\mathcal{E}(u,u) \geq 0\ \forall u \in \mathcal{F} \cup \widehat{\mathcal{F}}$).

Denote with $(G_\alpha)_{\alpha>0}$ the associated strongly continuous contraction resolvent on $\mathcal{H}$ (i.e. $(G_\alpha)_{\alpha>0}$ is a strongly continuous contraction resolvent on $\mathcal{H}$ such that