THE DIAMETRIC THEOREM IN HAMMING SPACES — OPTIMAL ANTICODES

Rudolf Ahlswede and Levon H. Khachatrian

Abstract

For a Hamming space $(\mathcal{X}_\alpha^n, d_H)$, the set of n–length words over the alphabet $\mathcal{X}_\alpha = \{0, 1, \dots, \alpha-1\}$ endowed with the distance d_H, which for two words $x^n = (x_1, \dots, x_n)$, $y^n = (y_1, \dots, y_n) \in \mathcal{X}_\alpha^n$ counts the number of different components, we determine the maximal cardinality of subsets with a prescribed diameter d or, in another language, anticodes with distance d. We refer to the result as Diametric Theorem.

In a sense anticodes are dual to codes, which have a prescribed *lower* bound on the pairwise distance. It is a hopeless task to determine their maximal sizes exactly.

We find it remarkable that the Diametric Theorem (for arbitrary α) can be derived from our recent Complete Intersection Theorem, which can be viewed as a Diametric Theorem (for $\alpha = 2$) in the restricted case, where all n–length words considered have exactly k ones.