Uniqueness of Gibbs states for quantum lattice systems

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Abstract

We prove uniqueness of Gibbs states for certain quantum lattice systems with unbounded spins. We use Dobrushin's uniqueness criterion. The necessary estimates for the Vasershtein distance between the corresponding one-point conditional distributios with boundary conditions differing only at one side, are obtained by proving a Log–Sobolev inequality on the infinite dimensional single spin (= loop) spaces. Some important classes of concrete examples to which all this applies are discussed.

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Contents

1 Introduction.

The purpose of this paper is to prove uniqueness of Gibbs states for certain quantum lattice systems with unbounded spins. It was proposed in [AH-K] to construct and study such states through a *Euclidean approach*. This approach transforms the problem of constructing such Gibbs states on the algebra of local observables into the problem of studying Gibbs measures for classical lattice systems, but with complicated infinite dimensional single spin spaces (as e.g. loop spaces). We shall describe this approach and the particular types of Hamiltonians considered here in more detail in Section 2 below. Section 3 is devoted to describe the corresponding Gibbsian formalism, i.e., the underlying local characteristics given by the heuristic Hamiltonian are defined rigorously through specifications determining the