Geometric aspects of finite and infinite dimensional Fleming-Viot processes

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Abstract

Some geometric aspects of Fleming-Viot processes are studied . In the finite-dimensional case we give an analytic proof that the Fleming-Viot process with parent-independent mutation is the only one that has a symmetrizing measure. This gives rise to a new proof that there is a unique symmetrizing measure for the infinite-dimensional Fleming-Viot process with parent-independent mutation. We show that the finite dimensional Fleming-Viot process with parent-independent mutation is a time changed Brownian motion with respect to a proper metric, and in some special cases is even a Brownian motion itself. A sequence of these finite-dimensional Fleming-Viot processes converges in probability to the infinite dimensional Fleming-Viot process. The main result for the infinite-dimensional Fleming-Viot process is on its intrinsic metric. We prove that the intrinsic metric is uniformly equivalent to the metric defined by the total variation norm.

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Contents

0 Introduction

This paper is on the geometry of reversible Fleming–Viot processes. This type of measure-valued diffusions and its corresponding analytic quantities have been studied intensively in recent years. For excellent survey articles we refer to [EK93] and [D93] and the references therein. For a more analytic approach based on the theory of Dirichlet forms we refer to recent joint work by the authors and B. Schmuland (see [ORS95]).

The purpose of this paper is threefold: first, we give a complete analytic proof for the characterization of reversible Fleming–Viot processes in finite dimensions (leading to