Dirichlet forms on infinite-dimensional "manifold-like" state spaces: a survey of recent results and some prospects for the future

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Dedicated to our admired colleague, teacher, and friend Prof. Masatoshi Fukushima on the occasion of his 60th birthday.

Abstract

We give a (to some extent pedagogical) survey on recent results about Dirichlet forms on infinite-dimensional "manifold-like" state spaces including path and loop spaces as well as spaces of measures. The latter are associated with interacting Fleming-Viot processes resp. infinite particle systems. Also some new results, further developing the Dirichlet form approach to infinite particle systems, are enclosed. Finally, a brief summary of other research activities in the theory of Dirichlet forms is given and some prospects for the future are indicated.

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1 Introduction

The purpose of this paper is to give a survey of recent results on Dirichlet forms and their associated Markov processes on infinite-dimensional "manifold-like" state spaces. At the same time we try to indicate directions of future research in this area. The selection of topics presented here is very much biased and concentrates on results mostly achieved by the author in collaboration with a number of colleagues. The exposition is to some extent pedagogical, adressing the non-expert reader, supressing technicalities as much as possible, and giving more emphasis to ideas and prospects rather than proofs. We start with the classical case of gradient-type Dirchlet forms on finite-dimensional manifolds in Section 2, summarizing the necessary modifications to go to infinite-dimensional situations. The latter are presented in the subsequent Sections 3 to 6 in the shape of three *model-case studies*. The first concerns gradient-type Dirichlet forms on Banach (or topological vector) spaces. Special emphasis is given here to the classical Wiener space (cf. Section 3). In the second and third model-case studies the underlying infinite-dimensional "manifolds" are *no longer flat*. More precisely, we consider Dirichlet forms on loop spaces over compact finite-dimensional Riemannian manifolds (cf. Section 4) resp. Dirichlet forms on state spaces of measures over a polish space (cf. Sections 5 and 6). The latter are related to Fleming-Viot processes resp.infinite particle systems. We emphasize that in Section 6 all underlying hard results are borrowed from [Y95], [Os95]. Our contribution here is only to provide the suitable geometric framework, i.e., we identify a natural underlying tangent bundle structure which, in particular, enables us to introduce more general Dirichlet forms than the ones studied in [Y95], [Os95] (e.g. we can allow non-constant diffusion coefficients). Thus, Section 6 contains really new results and should hence be of interest also for experts. Subsequently, in Section 7 we describe a recent application of gradient-type Dirichlet forms on Banach spaces (cf. Section 3) to the study of Gibbs states in statistical mechanics. Finally, we include a list of other topics presently being studied intensively within Dirichlet space theory. However, we do not claim any completeness of this list, neither w.r.t. the topics nor the scientists mentioned. On the contrary we would like to apologize at this point towards all people who should have been also mentioned or whose work should have been quoted. To keep the cited literature within bearable limits we mostly refer to the recent monographs [BH91], [MR92], [FOT94] and the references therein as well as the proceedings [FFGKRS93], [MRYa95].

Concerning the prospects for the future I do not wish to single out any special sub-branches in Dirichlet form theory that I personally think are of particular importance. I only want to express my sincere conviction that this theory and its applications bear a substantial amount of deep mathematics