2-partition-transitive tournaments

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March 22, 1996

Abstract

Given a tournament score sequence $s_1 \geq s_2 \geq \cdots \geq s_n$, we prove that there exists a tournament T on vertex set $\{1, 2, \ldots, n\}$ such that the degree of any vertex i is s_i and the subtournaments of T on both the even and the odd vertices are transitive in the given order. This means that i beats j whenever $i < j$ and $i \equiv j \pmod 2$.

For any score sequence, we give an algorithm to construct a tournament of the above form, i.e. it is transitive on evens and odds in the given order. This algorithm fixes half of the edges of the tournament and then is similar to the algorithm for constructing a tournament given its score sequence.

Another consequence provides asymptotics for the maximum number of edges in score unavoidable digraphs.

From a result of Ryser, it is possible to get from any tournament to this special tournament by a sequence of triangle reversals. We show that $n^2/2$ reversals are always enough and that in some cases $(1 - o(1))n^2/32$ are required. We also show that such a sequence of triangle reversals can be found in $O(n^2)$ time.